Traveling-Wave Superconducting Parametric Amplifiers

Peter Day, Byeong-Ho Eom, Rick Leduc

Jet Propulsion Laboratory, California Institute of Technology

Nikita Klimovich, Jonas Zmuidzinas

California Institute of Technology

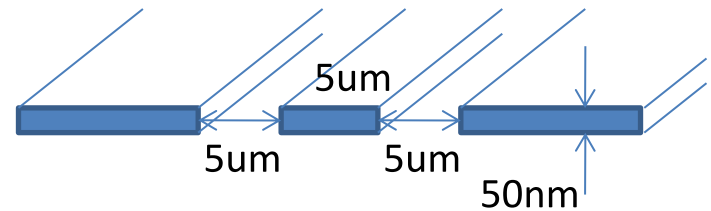
Jet Propulsion Laboratory
California Institute of Technology

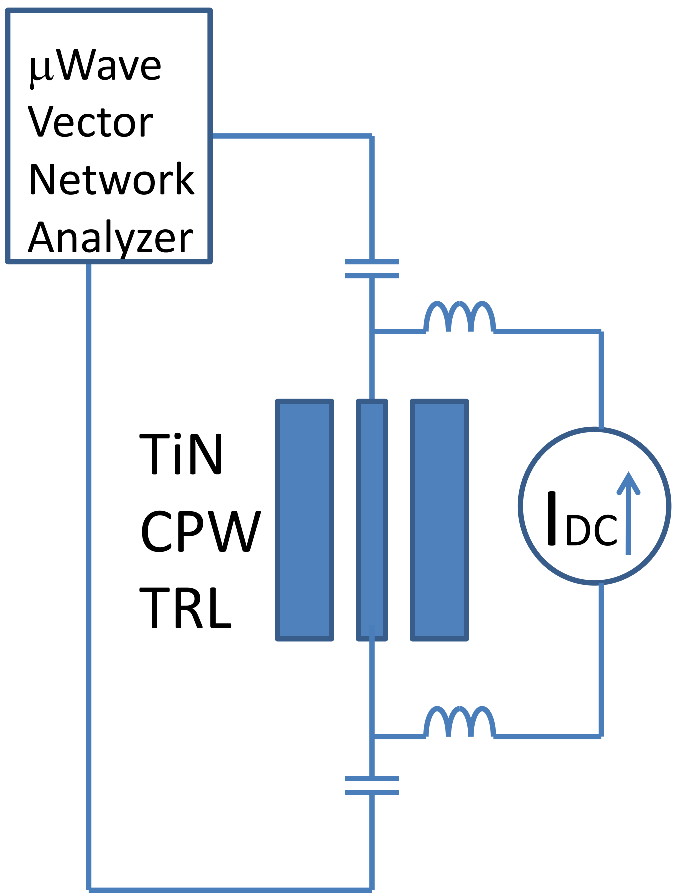
Outline

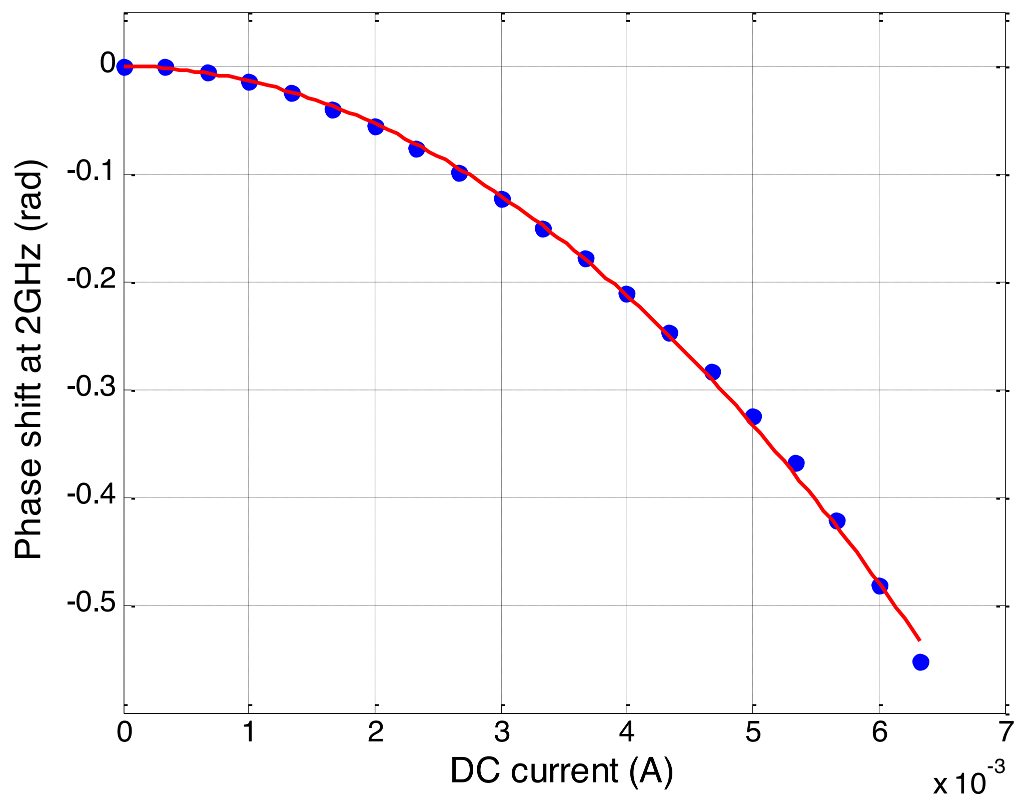
- Nonlinearity of kinetic inductance
  - Manifestation in superconducting resonators
- Kinetic Inductance Parametric Up-Converter (KPUP)
  - Multiplexable current sensor
- Kinetic inductance parametric amplifiers

Nonlinear kinetic inductance

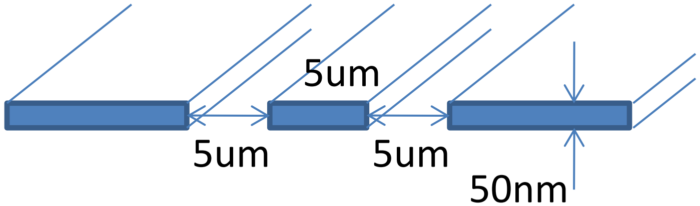
0.1 meter long CPW line





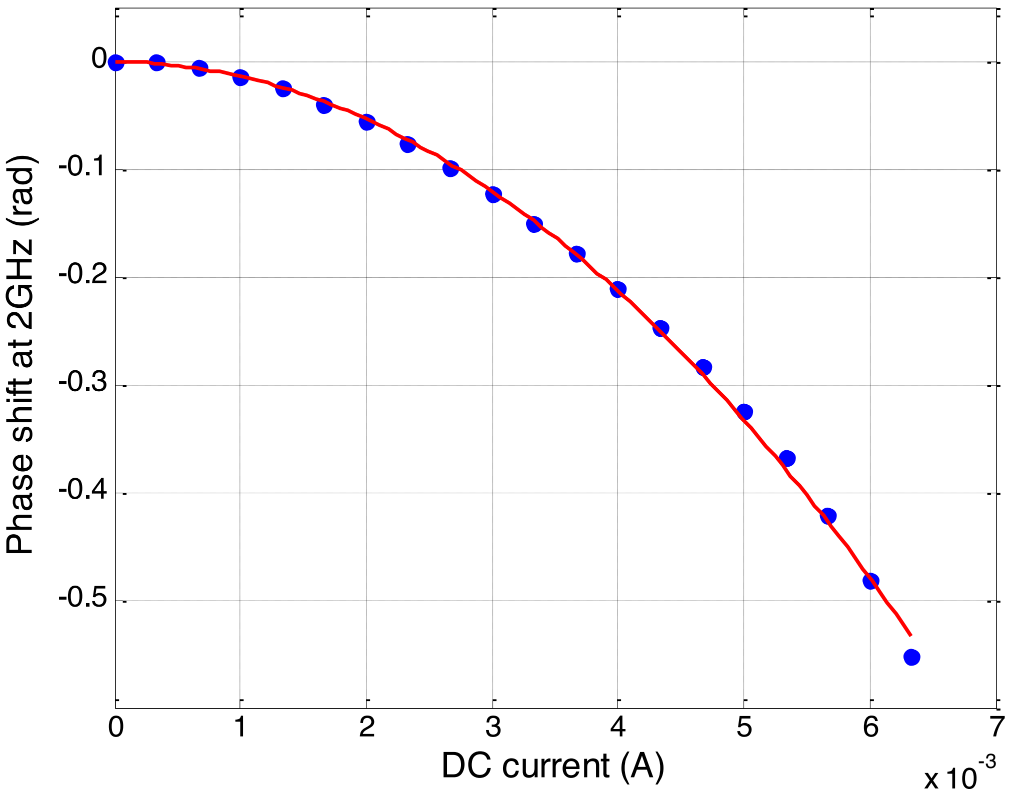


Nonlinear kinetic inductance



- $\Delta\theta \sim I^2$

$$L_k(I) = L_k(0)\left(1 + \frac{I^2}{I_*^2} + \ldots\right)$$



- Line length =0.1m

-> 21 radians

- $\Delta L/L(0) \sim 5\%$

Kinetic Inductance Non-linearity

- Ginsberg-Landau theory
  - suppression of superconducting order parameter by superfluid velocity

$$|\psi|^2 = \psi_\infty^2 \left[1 - \left(\frac{\xi m^* v_s}{\hbar}\right)^2\right]$$

$$\frac{n_s^*}{n_{s,0}^*} = 1 - \frac{1}{2}\frac{\mu_0 \lambda_{eff}^2 J_s^2}{\mu_0 H_c^2}$$

Supercurrent kinetic energy

Condensation energy

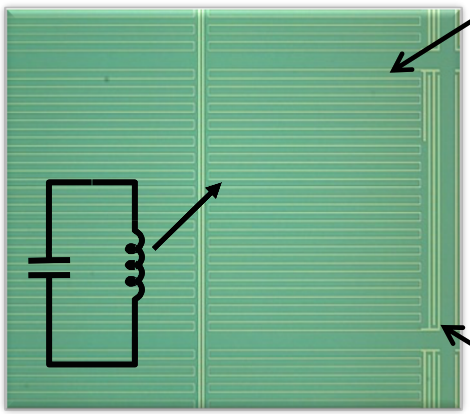
$$E_{pair} = 2N_0\Delta^2 V$$

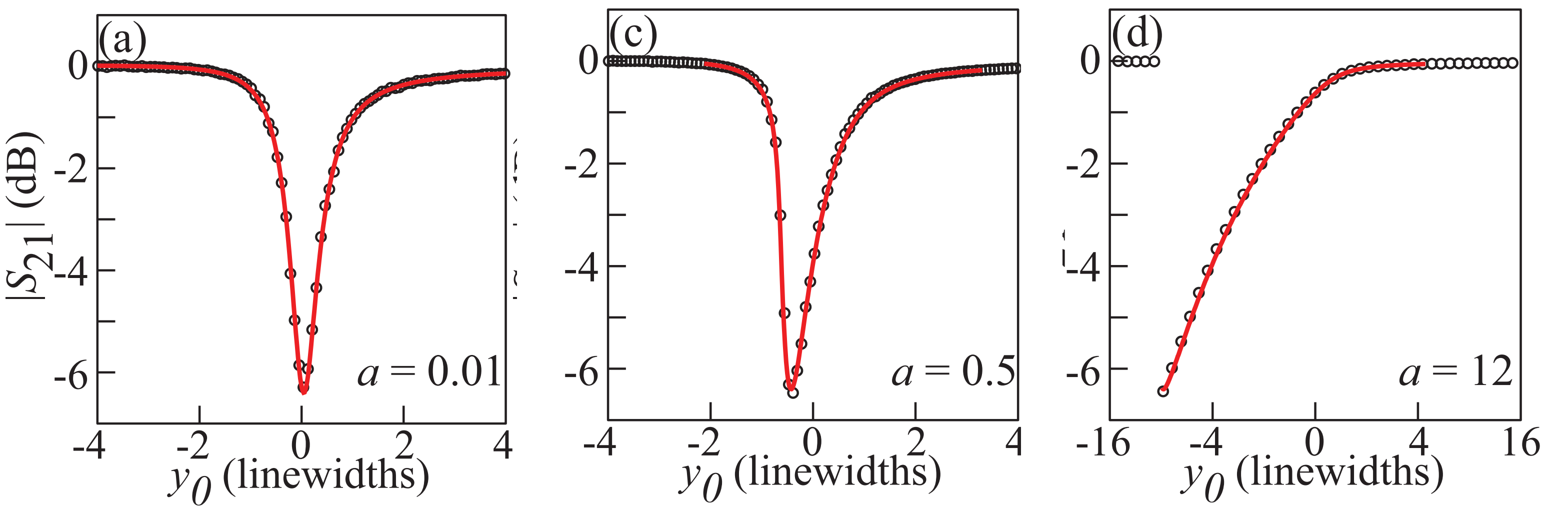
$$\frac{\delta L_s}{L_s} = \frac{\delta\lambda_{eff}}{\lambda_{eff}} = \frac{1}{2}\frac{\delta n_s^*}{n_{s,0}^*}$$

- Nonlinearity is large for materials with large λ
  - High normal state resistivity, eg. Nitrides TiN, NbTiN, …
- Δn_s is not associated with quasiparticle creation

TiN Resonator measurements

- Upward frequency sweeps

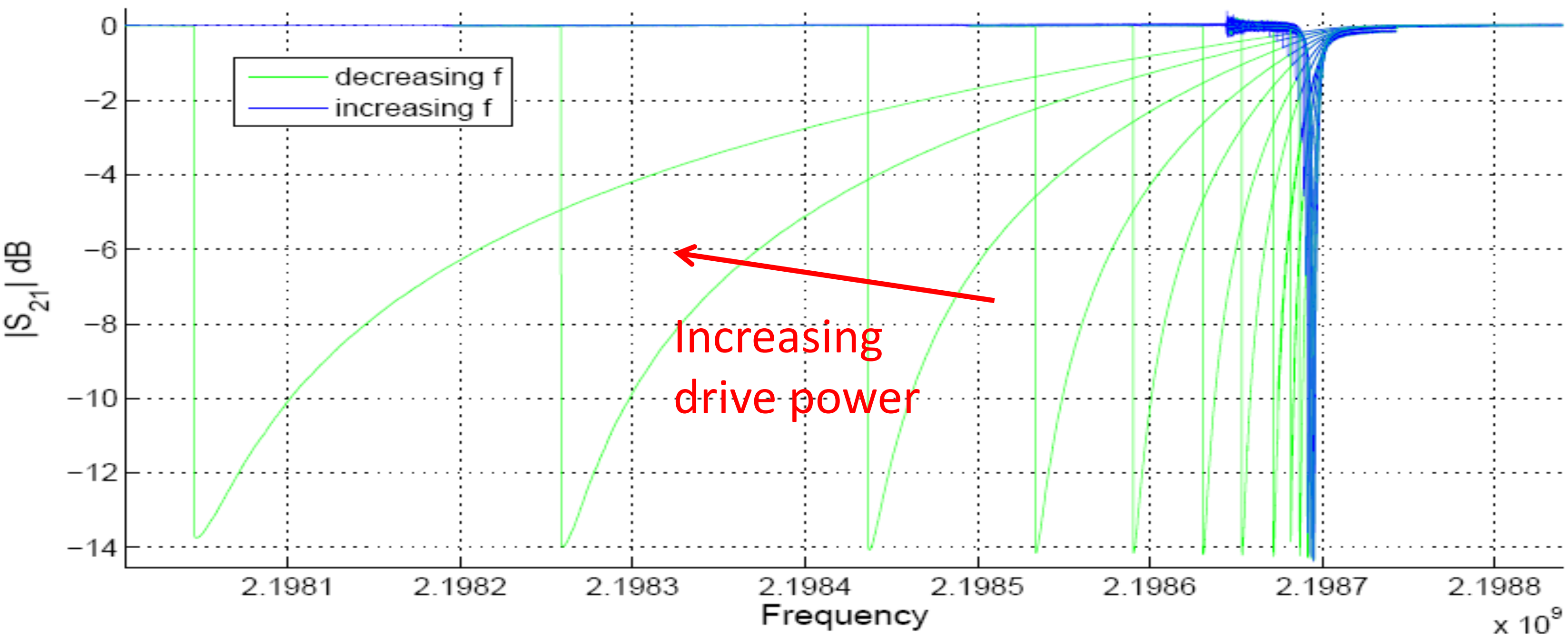




Increasing drive power

Nonlinear “Duffing” oscillator

- Resonance frequency depends on resonator current
- Hysteretic resonance curves:



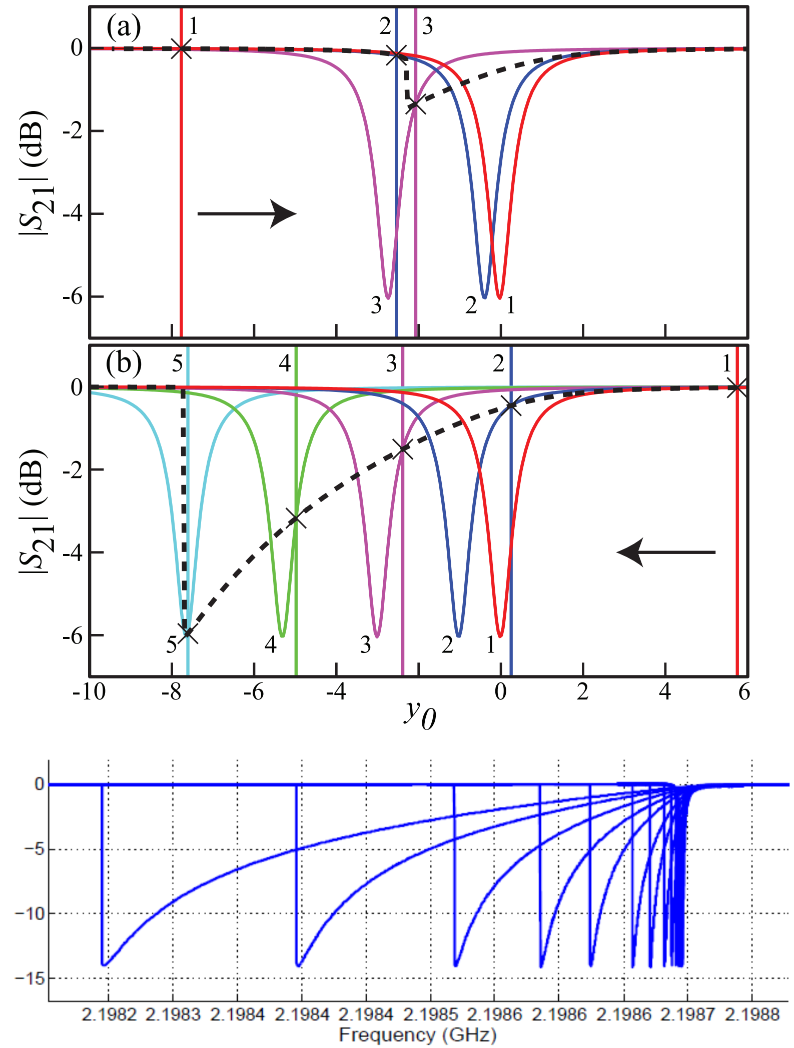
Non-linear resonator model

$$\delta f / f_0 = -k_f I_{res}^2$$
$$\delta Q_i^{-1} = k_Q I_{res}^2$$

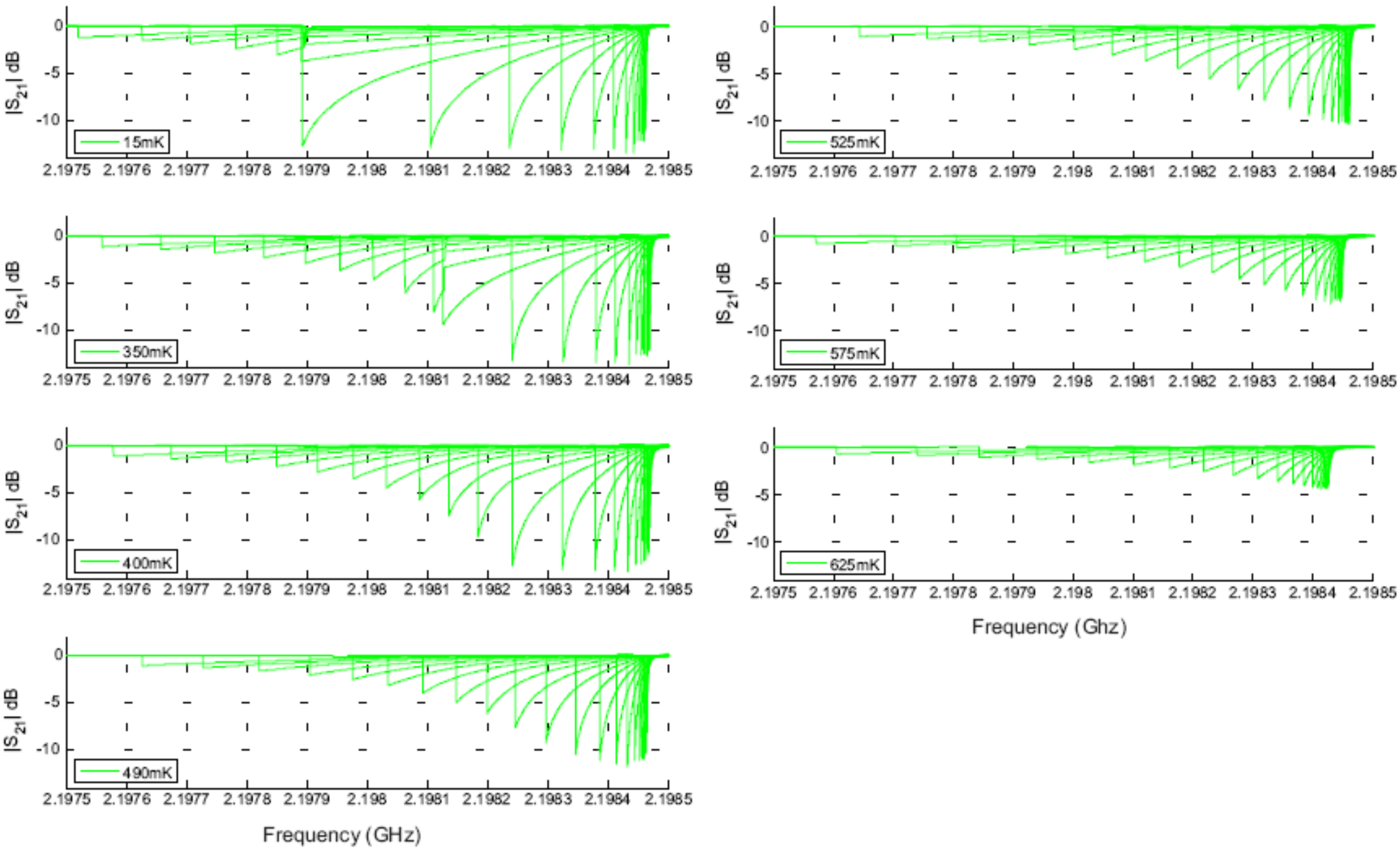
$$S_{21}(f, I_{res}) = 1 - \frac{Q_r / Q_c}{1 + 2iQ_r[f_{r,0} - \delta f_r(I_{res}) - f] / f_0}$$

$$I_{res}^2 = \frac{Q_c |1 - S_{21}|^2 P_{feedline}}{Z_0}$$

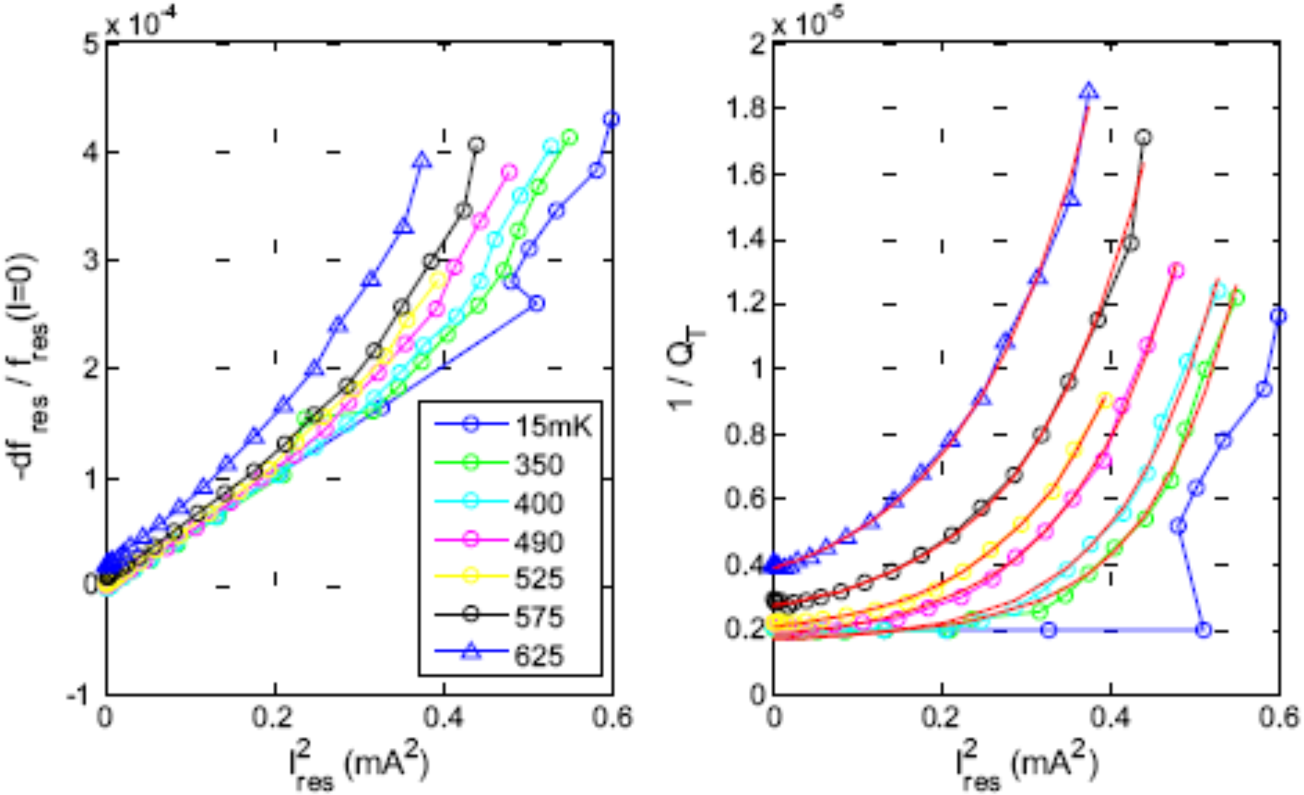
- Hysteresis past a critical drive power
- Determines contribution of amplifier to detector noise
- No nonlinear dissipation (under some conditions) - $k_Q = 0$!



Measurements at elevated T



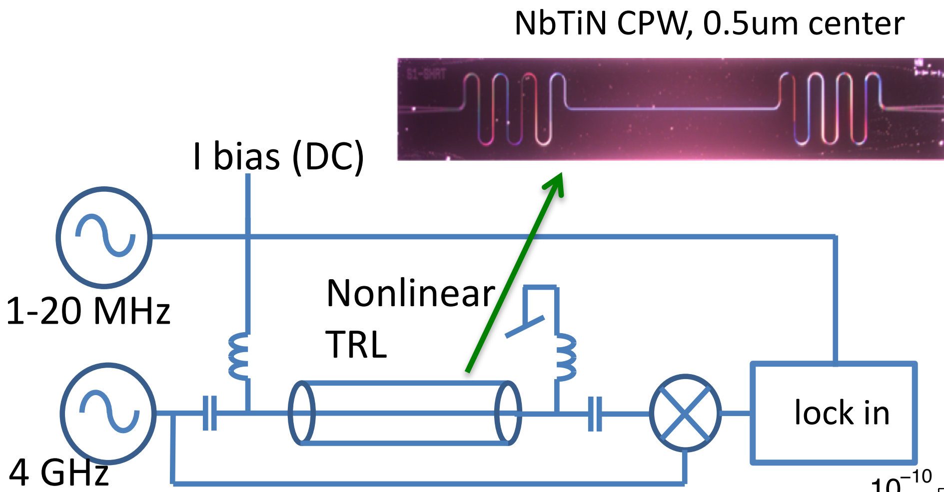
Measurements at elevated T

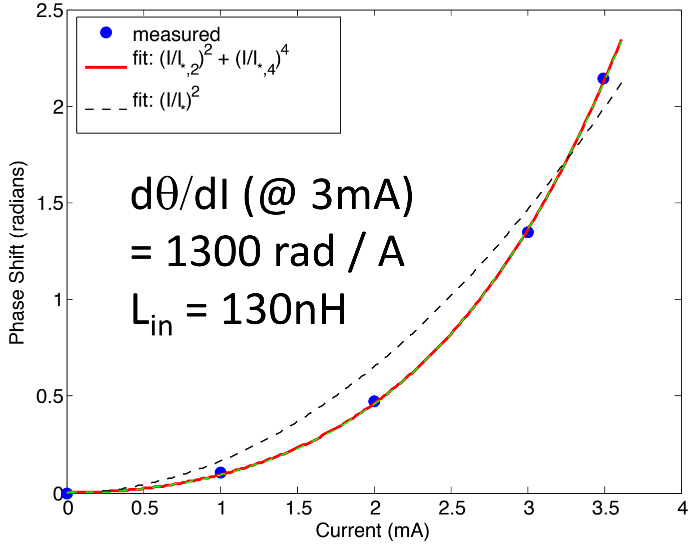


- Regime with low (zero?) nonlinear dissipation had not be accessed in previous attempts to use K.I. nonlinearity for device applications

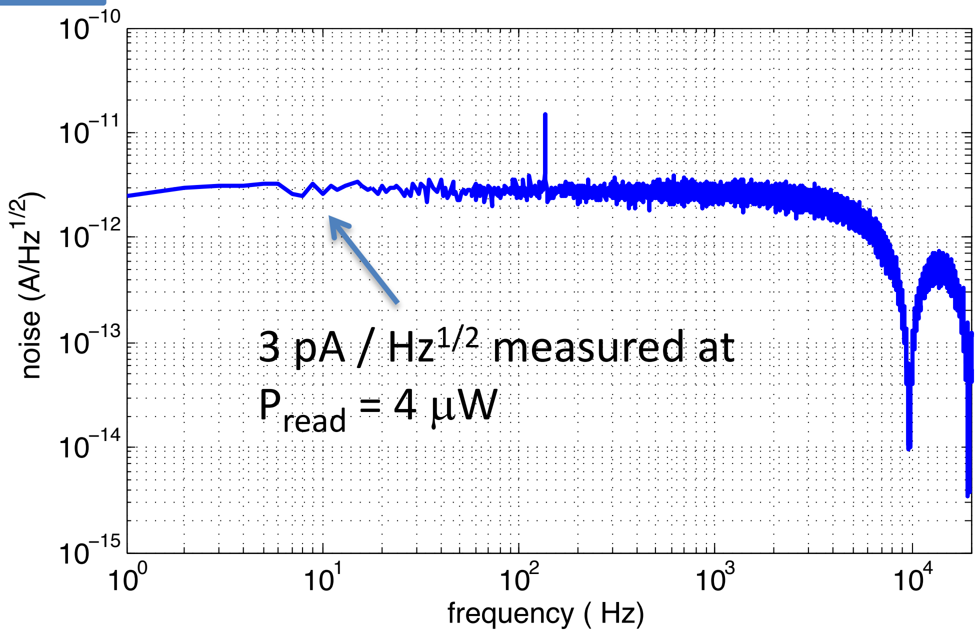
Kinetic Inductance Parametric Up-converter (KPUP)

Kinetic Inductance current sensor

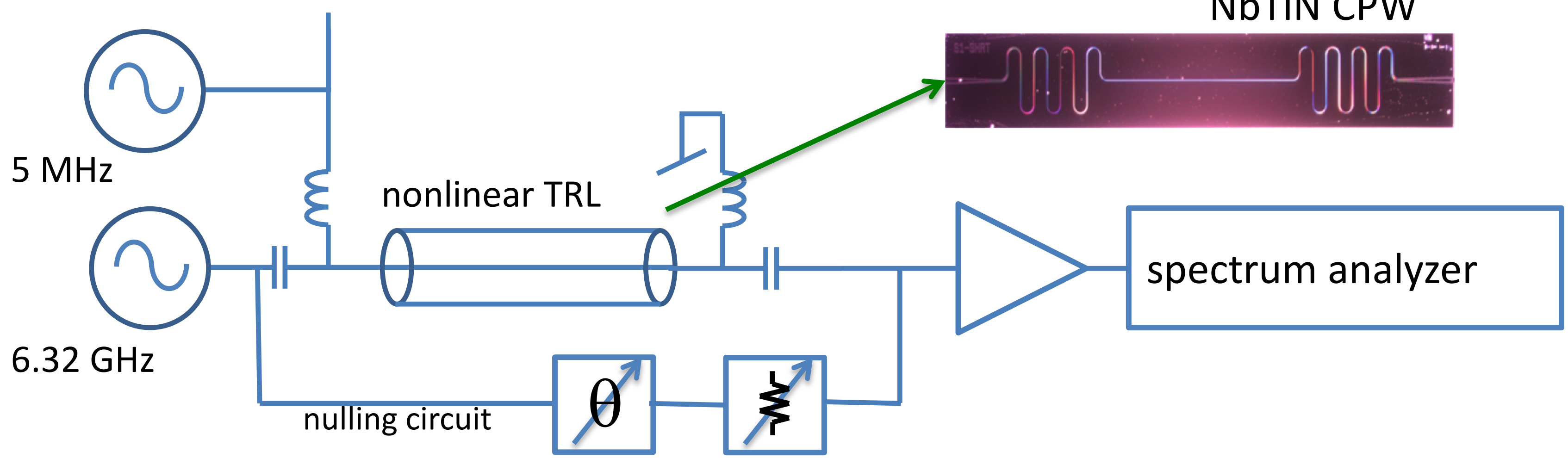




- $$(S_\theta^{amp})^{1/2} = \sqrt{\frac{k_b T_N}{2P_{read}}}$$
- for P_{read} = 1μW, T_N = 5K,
  – > $(S_I^{amp})^{1/2}$ ~ 6 pA / Hz$^{1/2}$



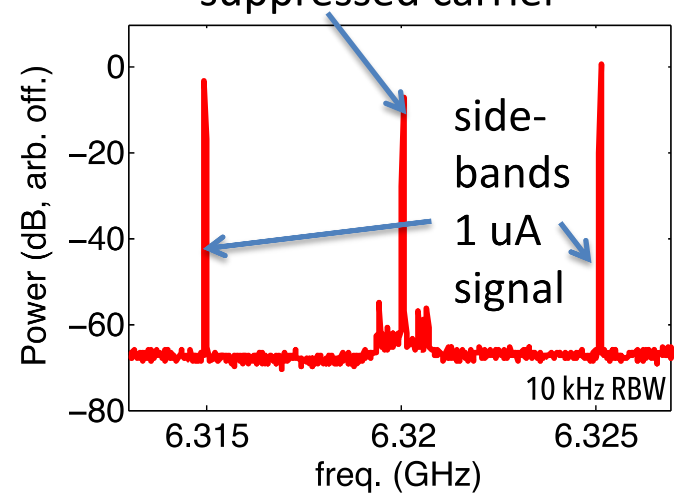
Up-conversion via kinetic inductance nonlinearity



- Sidebands produced through phase modulation:

$$A e^{i\omega_c t + i\xi I \sin \Omega t} \approx$$

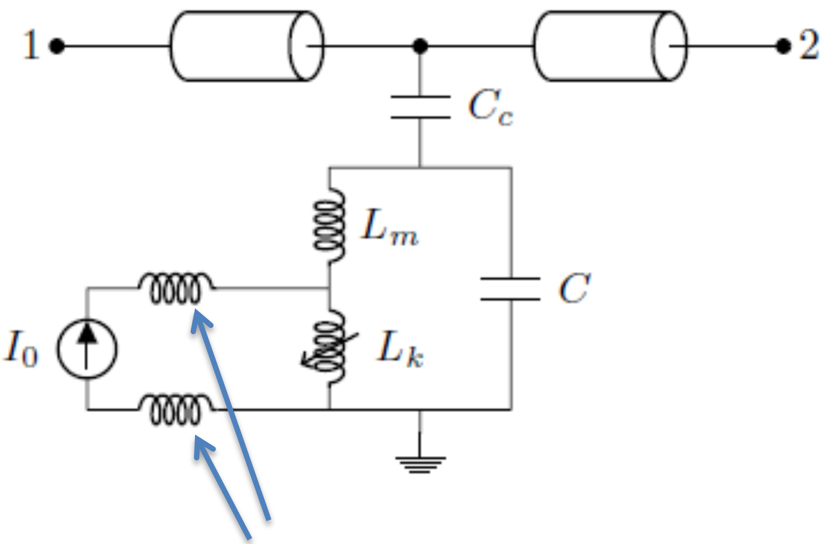
$$A e^{i\omega_c t} + \frac{A\xi I}{2} e^{i(\omega_c+\Omega)t} - \frac{A\xi I}{2} e^{i(\omega_c-\Omega)t}$$



KPUP device concept

- low frequency current modulates resonator frequency
- compared to non-resonant TRL:
  - phase response increased by Q
  - but max readout power decreased by Q^2
  - smaller input inductance

input / output microwave line

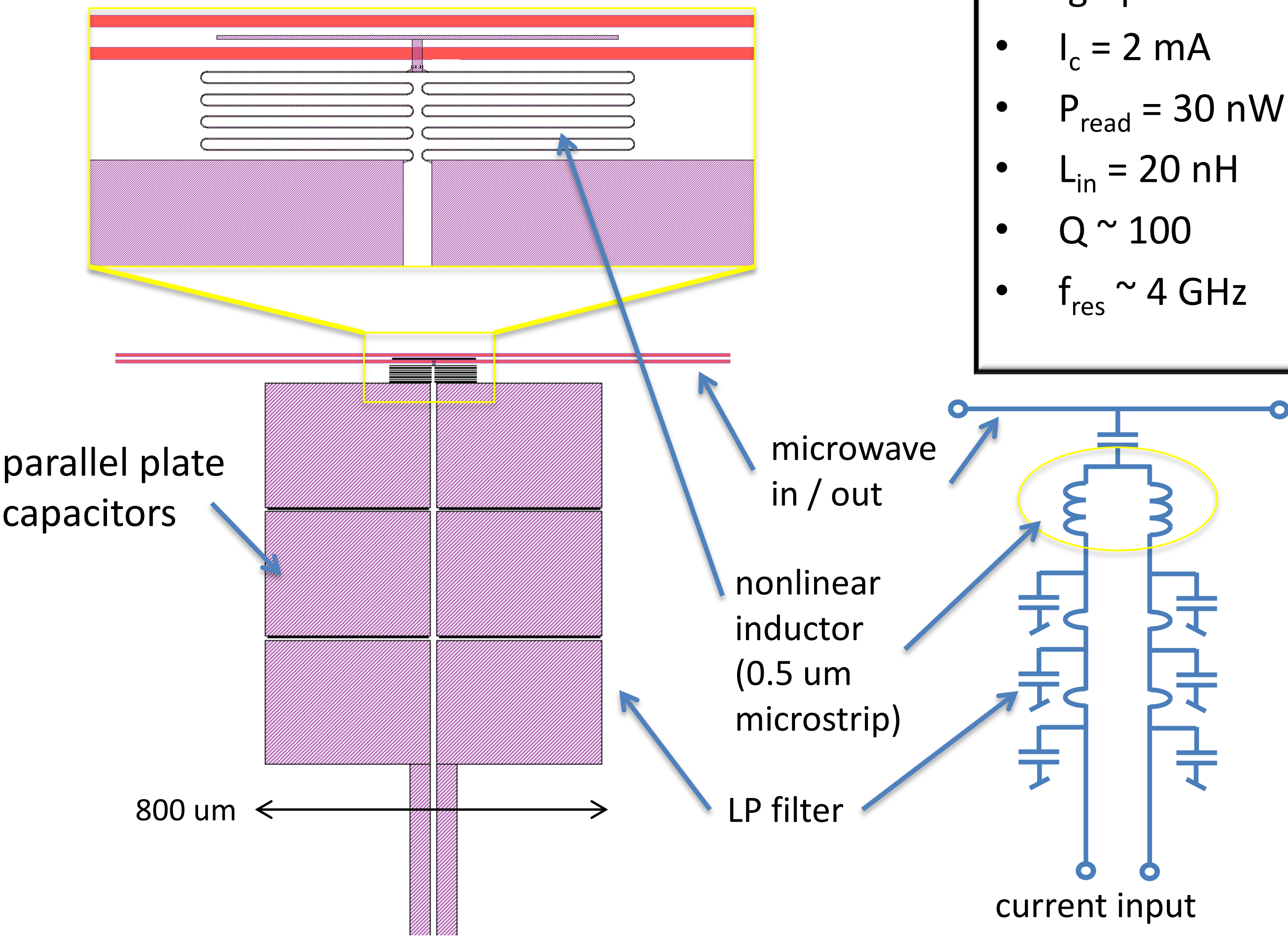


low pass filters

KPUP design

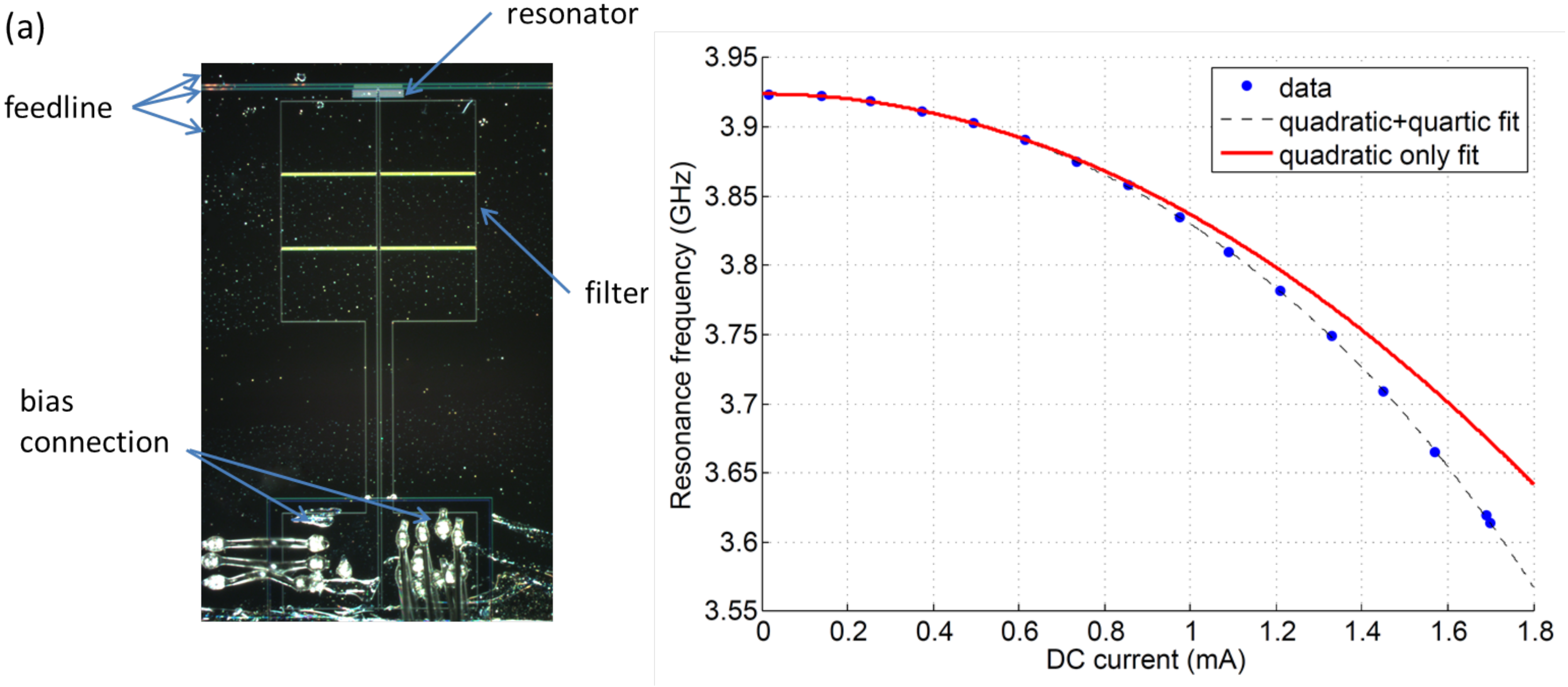
Design parameters:

- I_c = 2 mA
- P_{read} = 30 nW
- L_{in} = 20 nH
- Q ~ 100
- f_{res} ~ 4 GHz



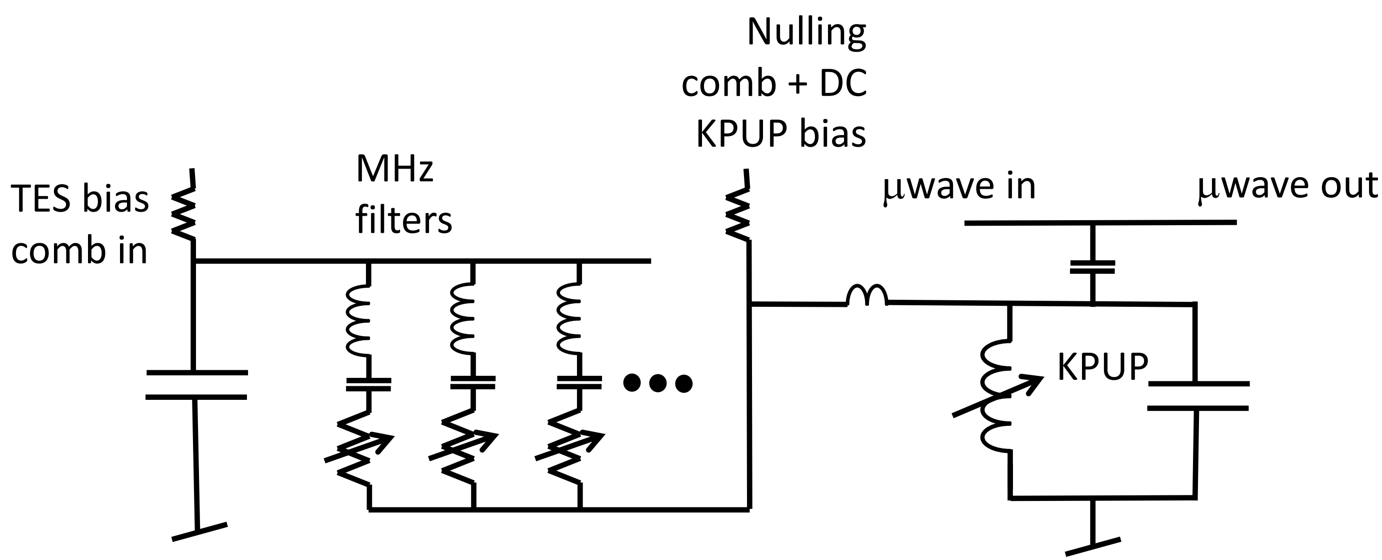
Frequency shift measurement

- Δf/f ~ 0.1 – no change in Q



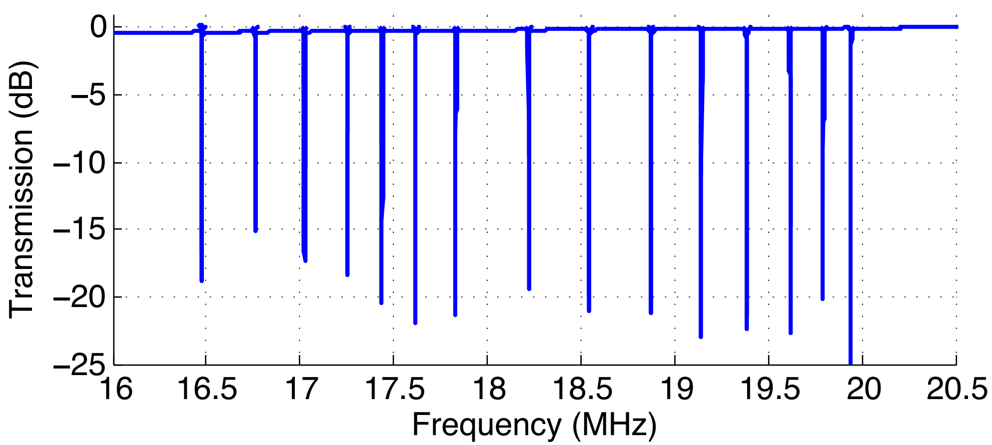
Application: TES readout circuit

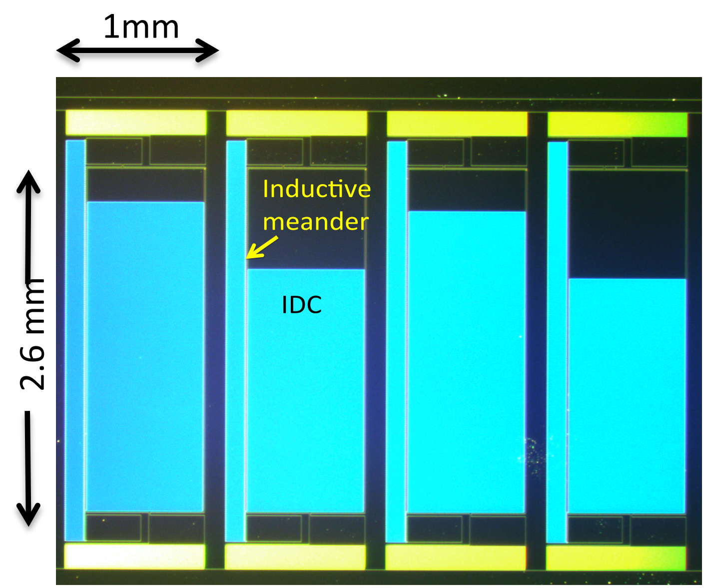
- Several TES signals combined at input to one KPUP
- Similar to circuit used for AC SQUID MUX
- Each signal creates a sideband on the microwave carrier

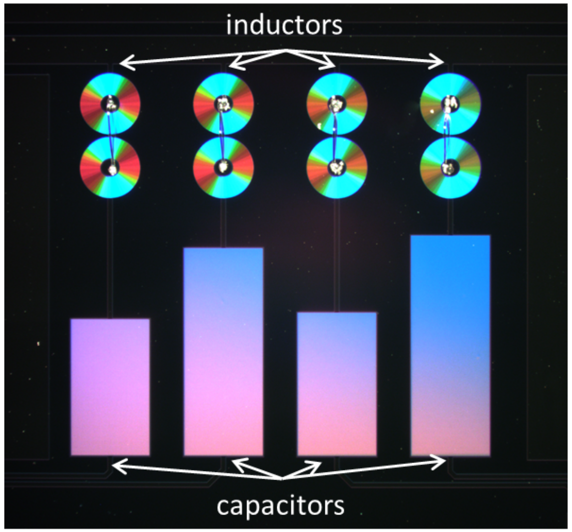


AC first stage MUX

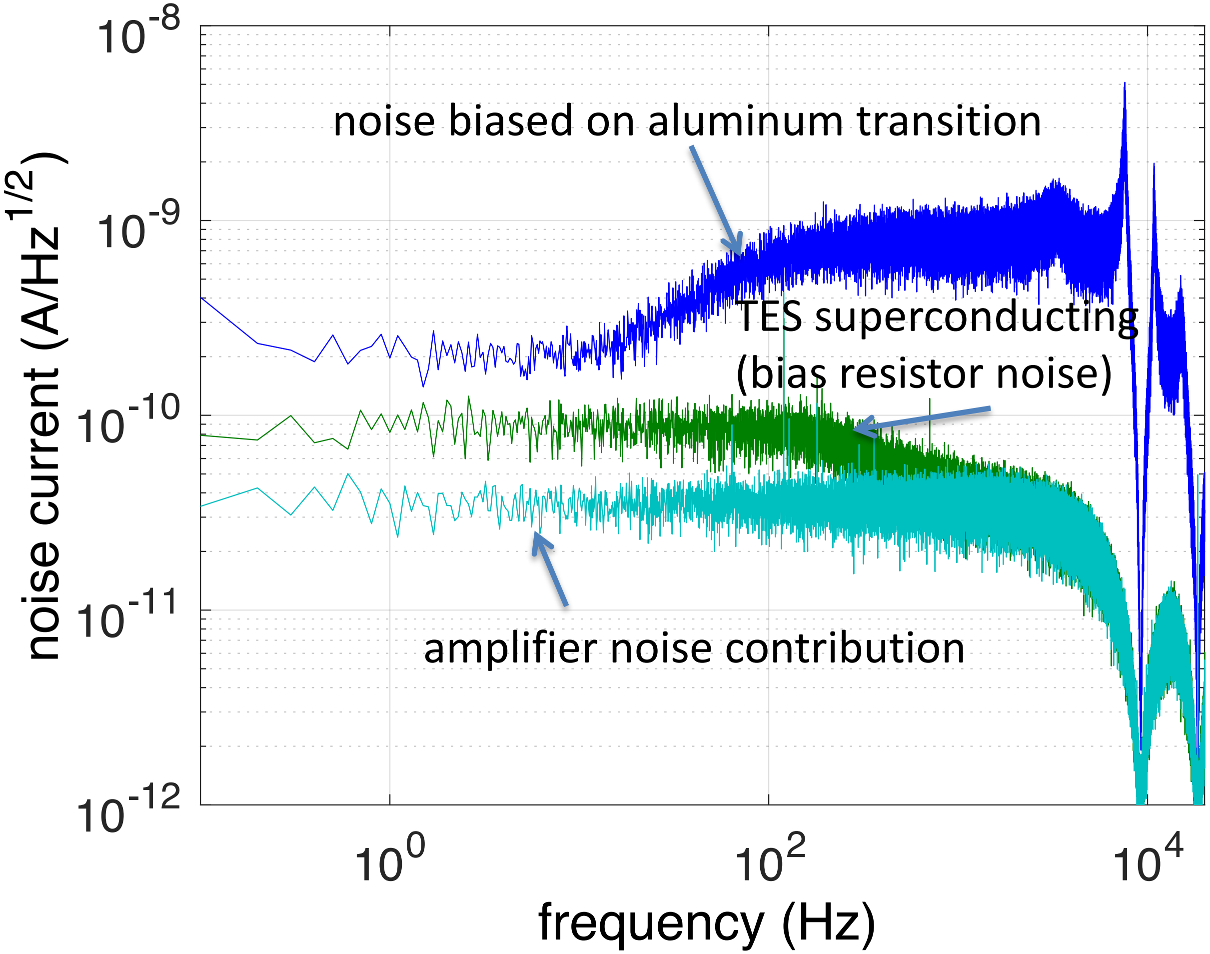
- L-C filters
  - single layer NbTiN
  - interdigital capacitors
  - inductance is mainly kinetic
  - L ~ 4 μH, C ~ 20 pF
  - new version uses spiral inductors





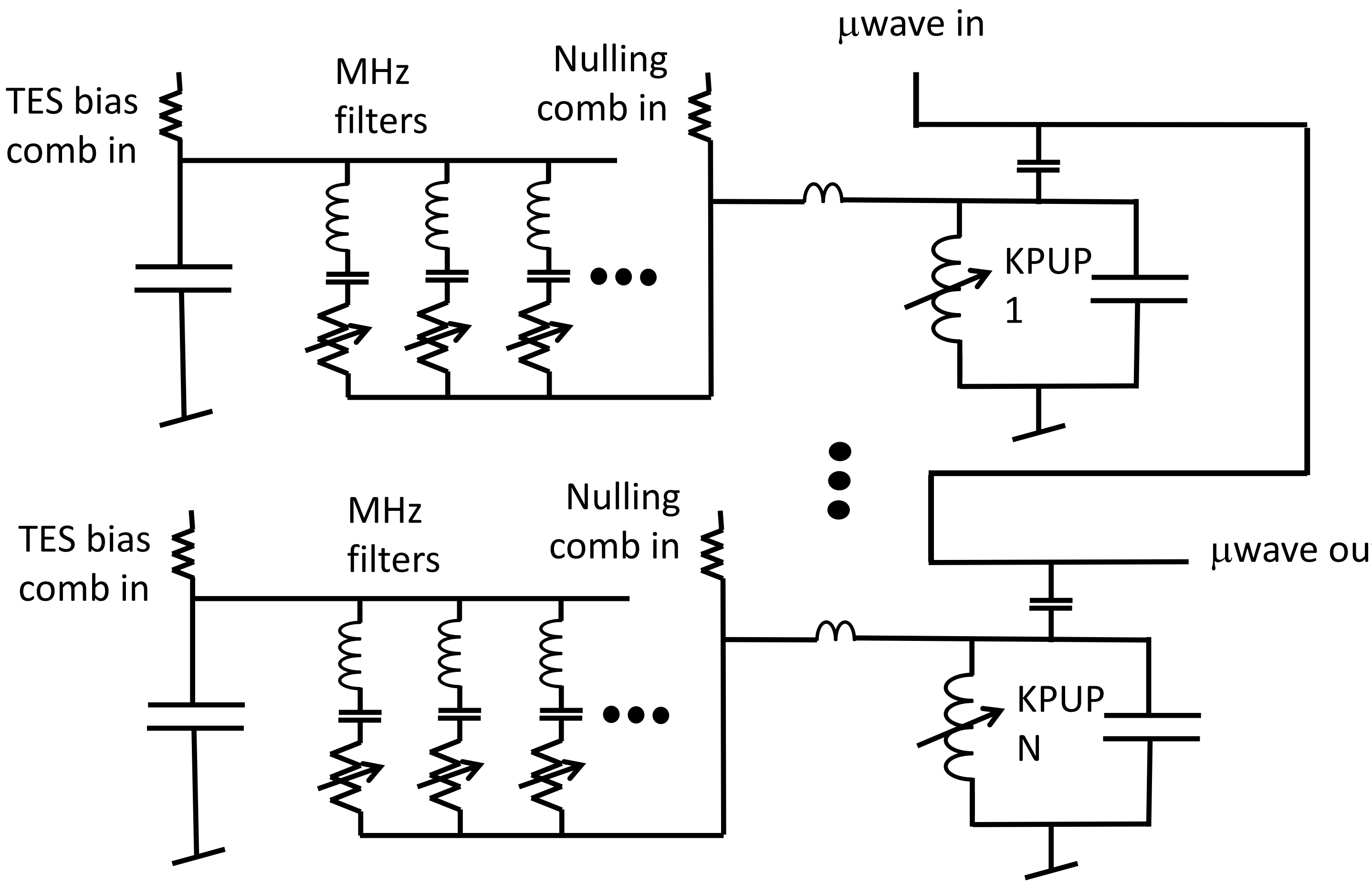


Noise



noise current (A/Hz^1/2)
10-8
10-9
10-10
10-11
10-12
noise biased on aluminum transition
TES superconducting
(bias resistor noise)
amplifier noise contribution
10^0
10^2
10^4
frequency (Hz)

MUX circuit



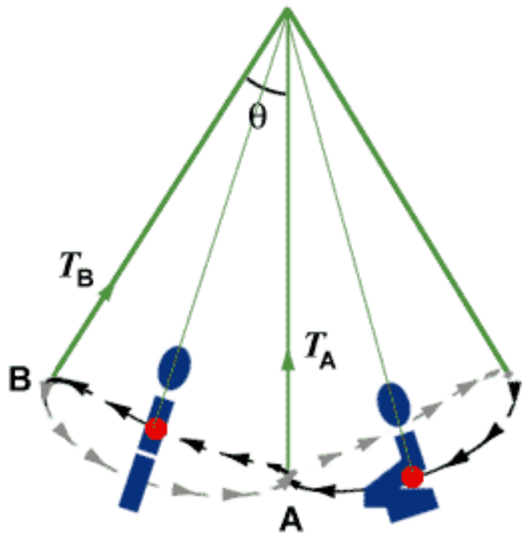
- Two-level multiplexing circuit
- First stage is similar to FDM SQUID mux

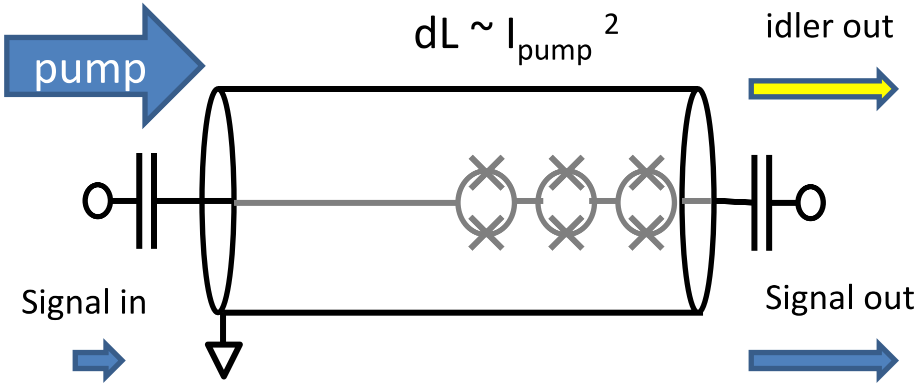
Compared to SQUID array

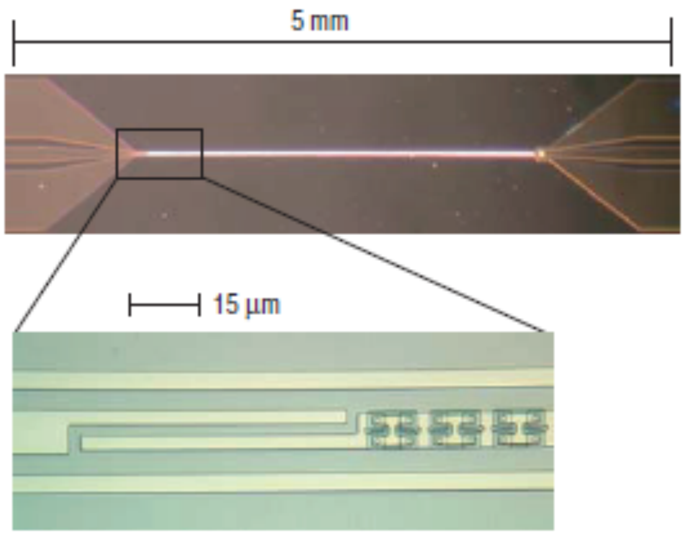
- Input inductance is somewhat smaller ~ 19 nH
  - compared to ~100 nH for SQUID array
  - could be decreased further
- Bias power is ~ 30 nW
  - Only a fraction of that is dissipated in KPUP
  - KPUP can be located on cold stage
  - Wiring inductance between TES and KPUP can be small
- Microwave readout allows for multiplexing of several KPUPs
- Output is monotonic, rather than periodic

Kinetic Inductance Traveling-wave (KIT) Paramp

Parametric amplifiers



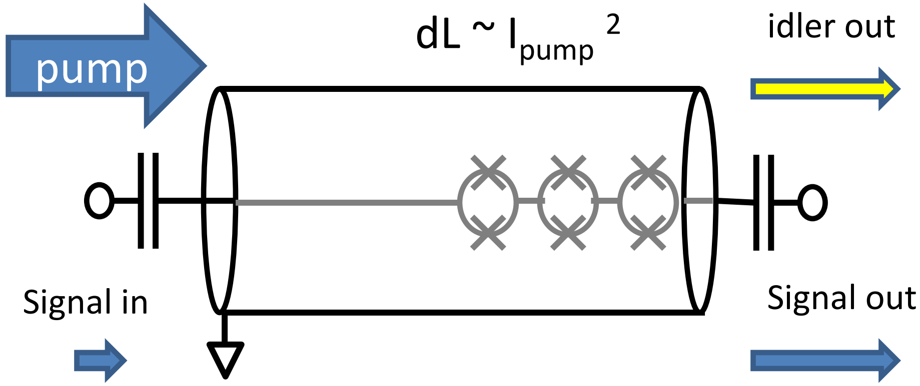


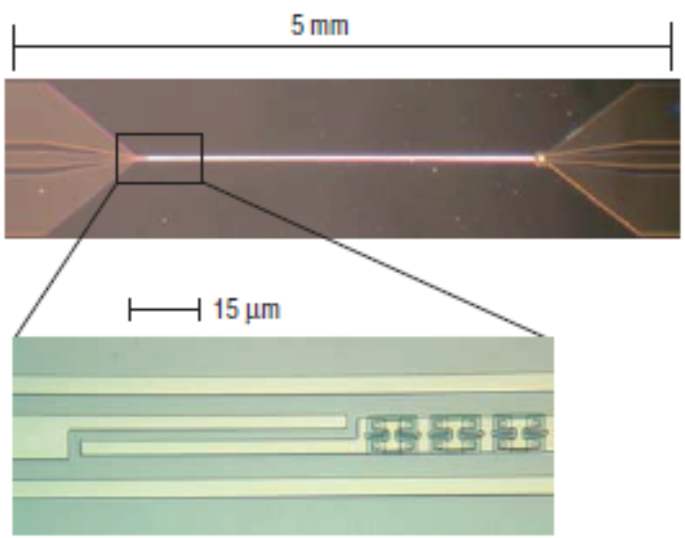


NIST Josephson Parametric Amplifier

Parametric amplifiers

- Nonlinearity
  - Response to strong pump tone
  - Transfer of energy from pump to signal
  - Amplification
  - Produce idler tone
  - $f_p = f_s + f_i$
  - or $2f_p = f_s + f_i$
- Purely reactive nonlinearity
  - *Need not add noise* (beyond QM requirement)

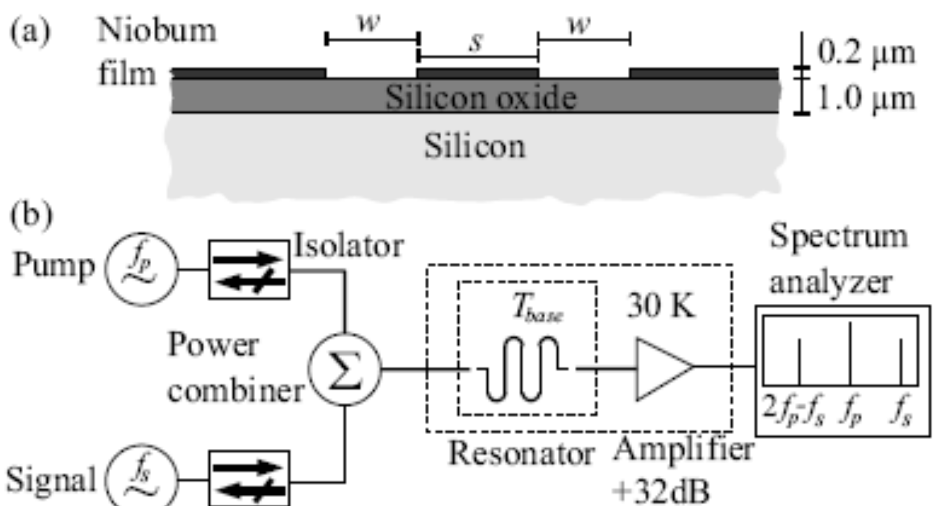


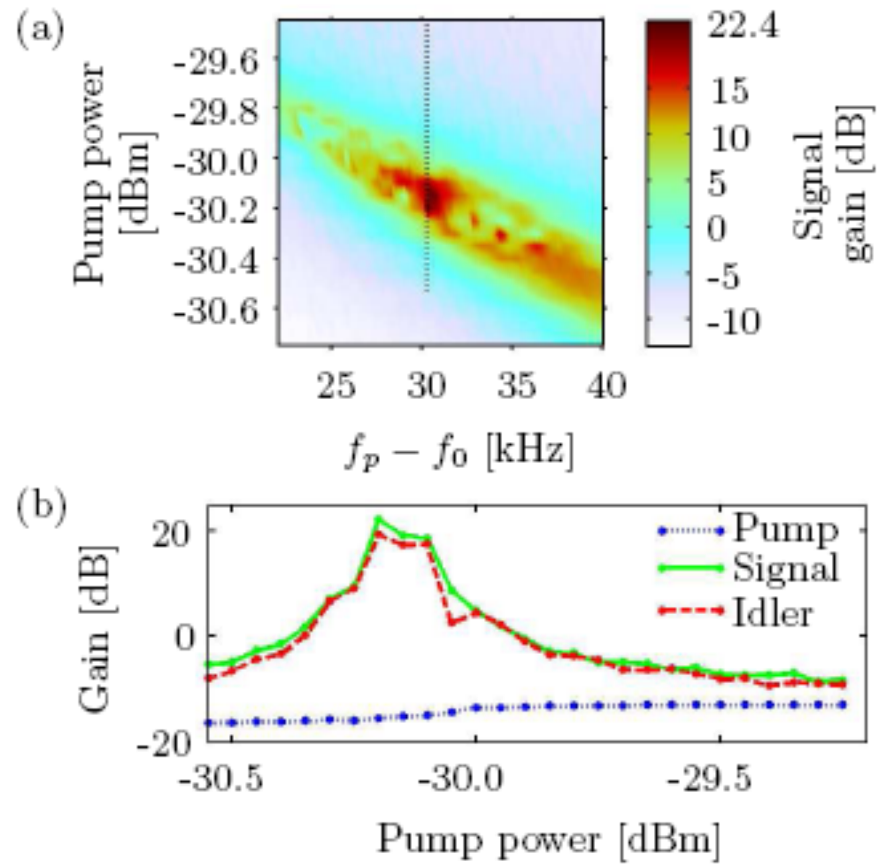


NIST Josephson Parametric Amplifier

Kinetic inductance cavity para-amp

- Tholen et al. (2007)
- Nb CPW resonator



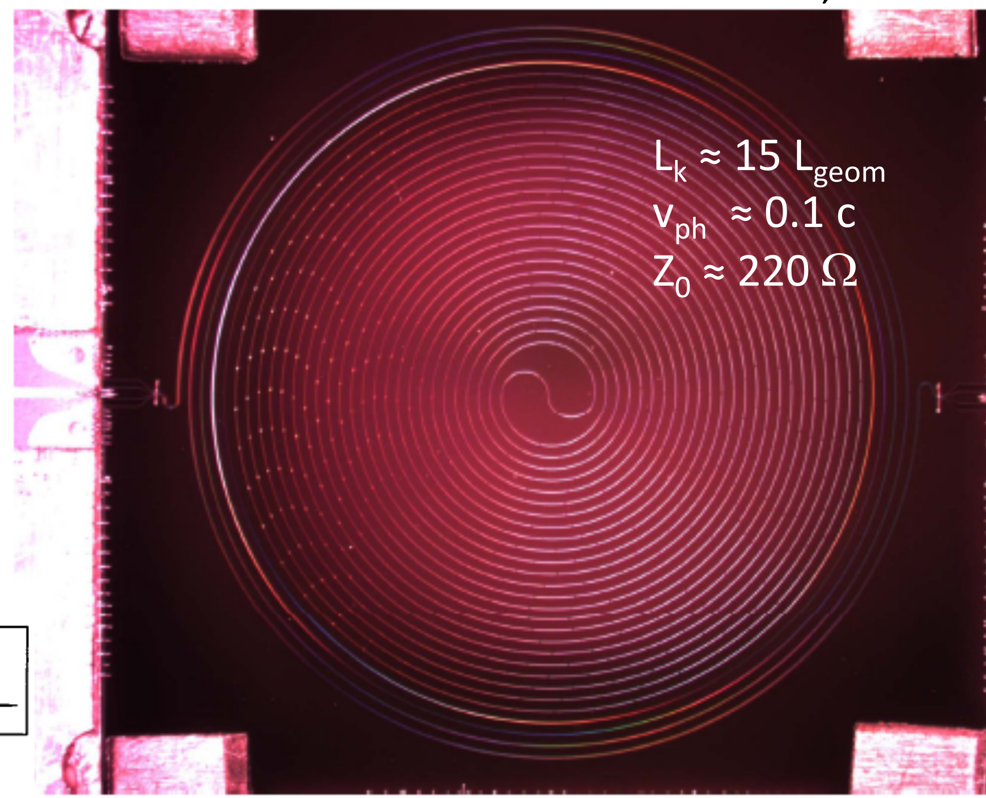


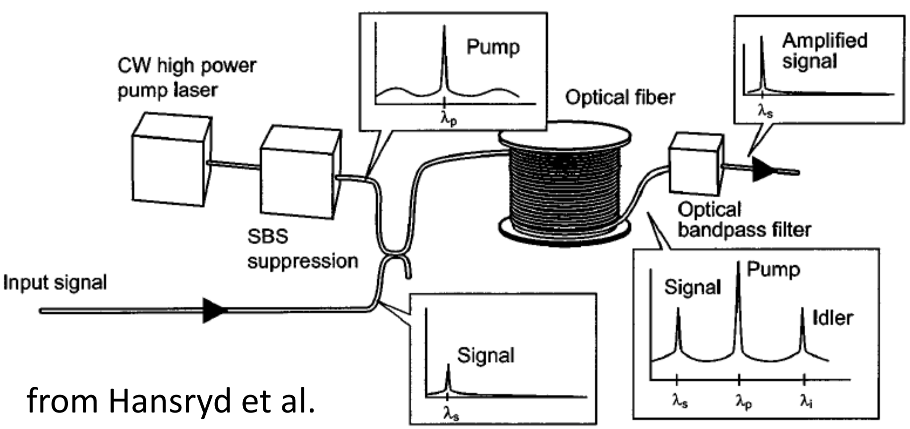
- Noise not reported

KIT paramp ver 1.0

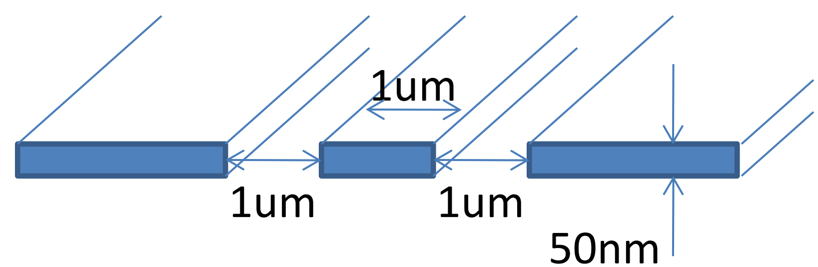
- Long nonlinear transmission line
  - Single layer of TiN or NbTiN
  - 0.8 m CPW length
  - Tapers at input, output match 50 ohms
- Analogous to visible frequency fiber optic paramp
  - Traveling wave design
  - Broadband

Eom et al, 2013





from Hansryd et al. (2002)

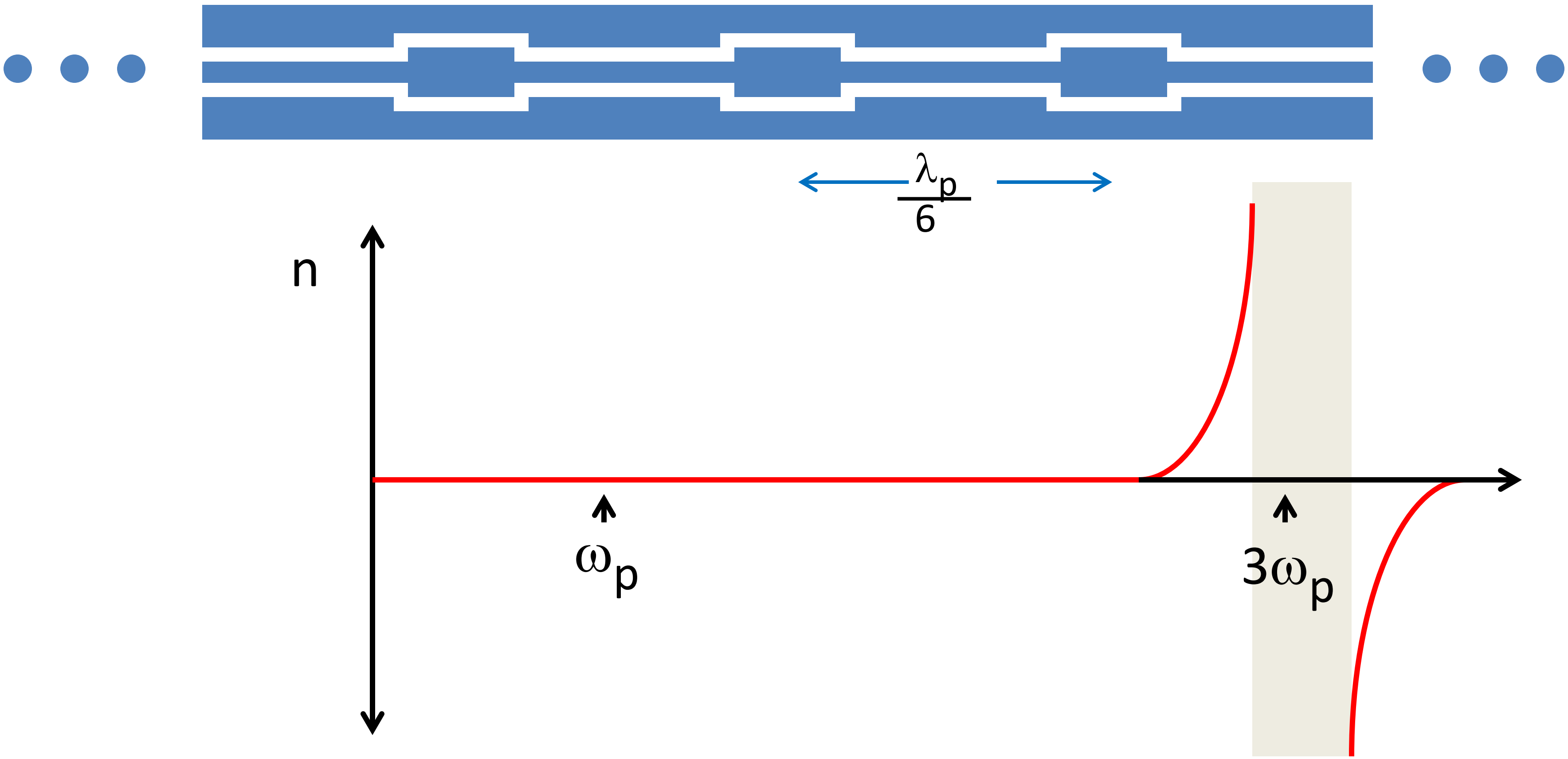


Traveling-wave amplifier design challenges

- Phase matching
  - Need to maintain phase relation between signal, pump, idler
  - Dispersion results in phase slippage between pump, signal and idler
  - Superconducting TRL nearly dispersionless for $f << f_{gap}$
- Non-linearity has a dispersive effect
  - Self Phase Modulation (SPM), Cross Phase Modulation (XPM)
- Harmonic generation is phase matched (and efficient) process in a dispersionless nonlinear TRL
  - Depletes pump power before useable gain

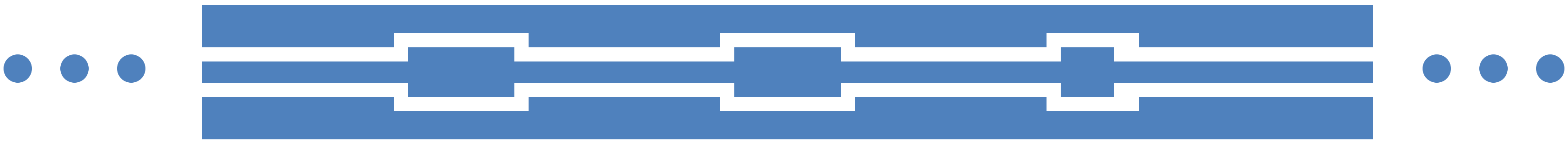
Dispersion Engineering

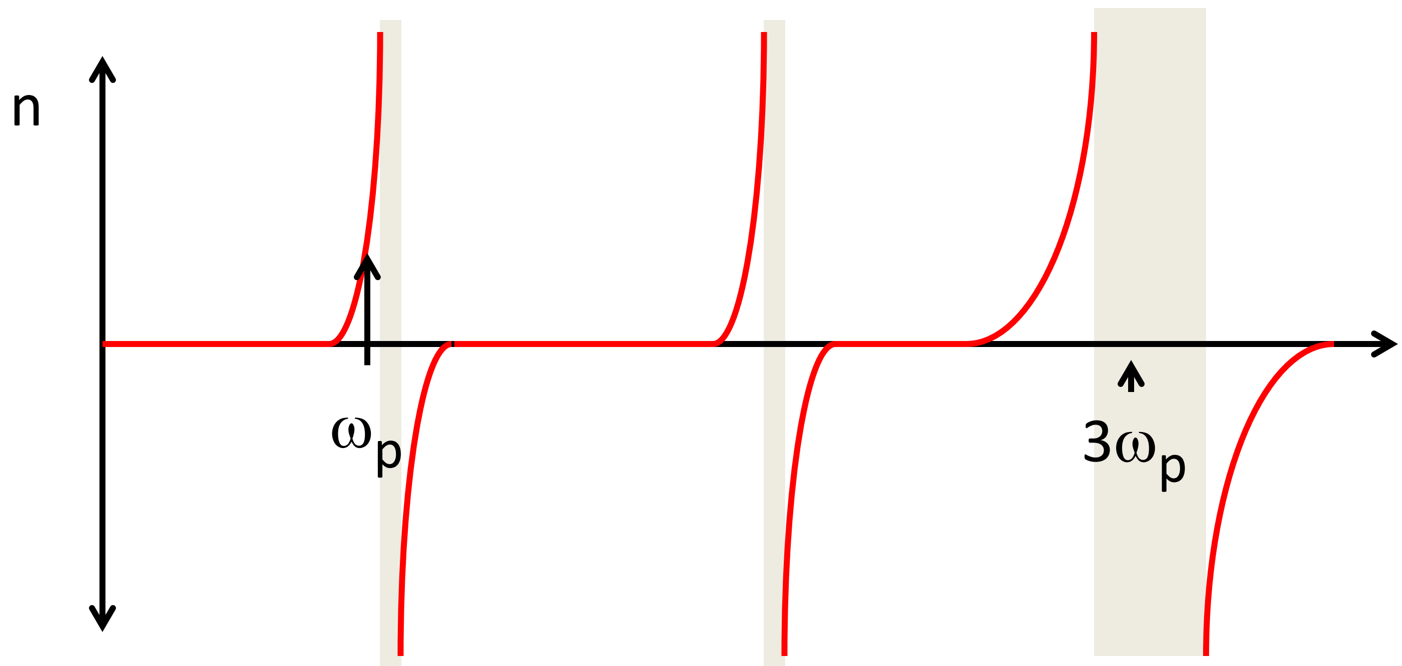
- Periodic loading to produce bandgap at $3\omega_p$



Achieving phase match

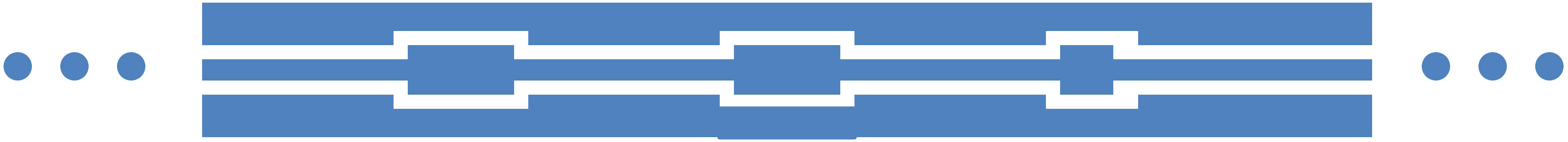
- Also use dispersion to cancel nonlinear phase slippage

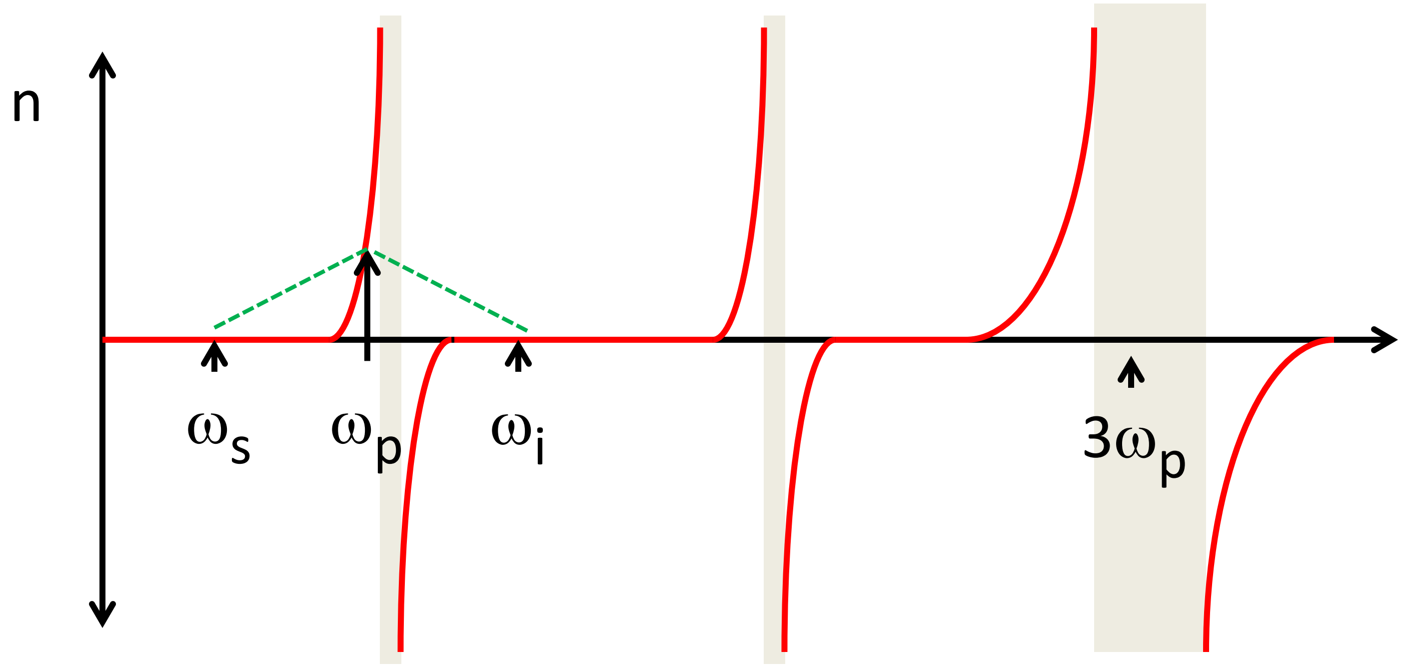




Achieving phase match

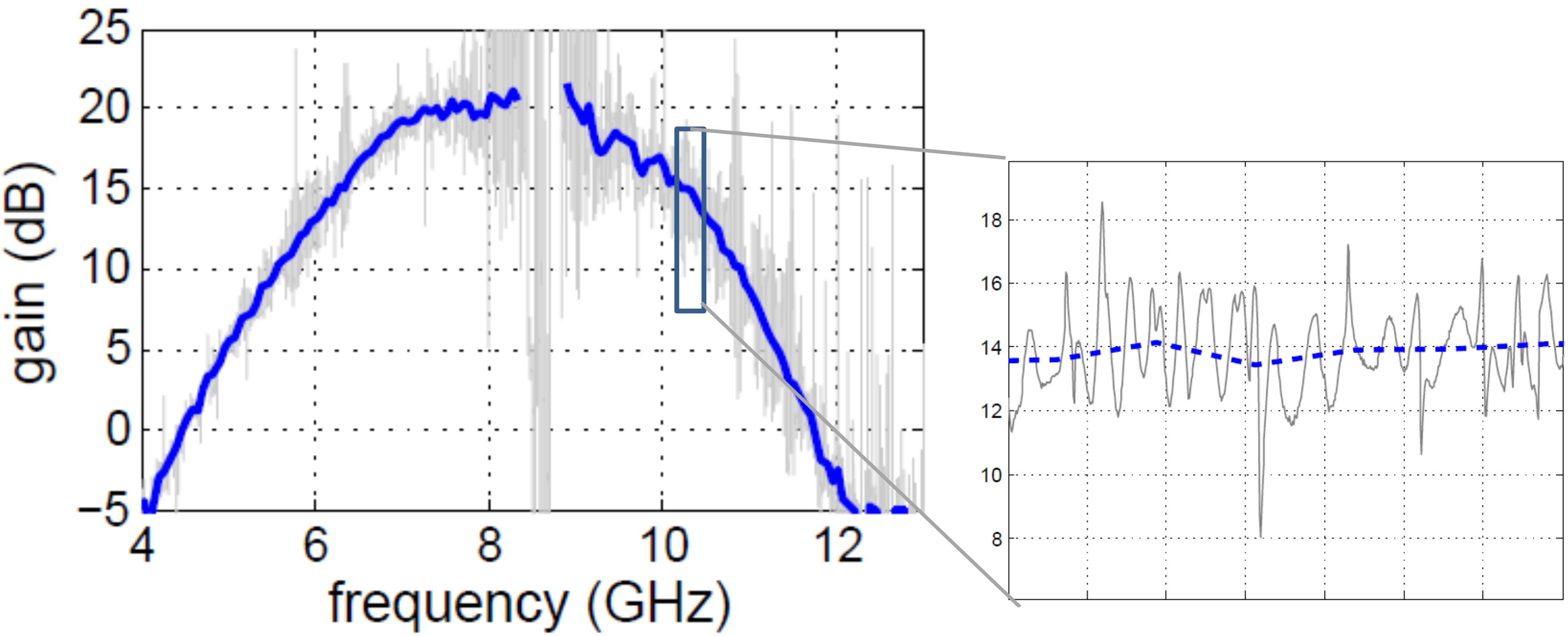
- Also use dispersion to cancel nonlinear phase slippage





V1.0 Paramp gain

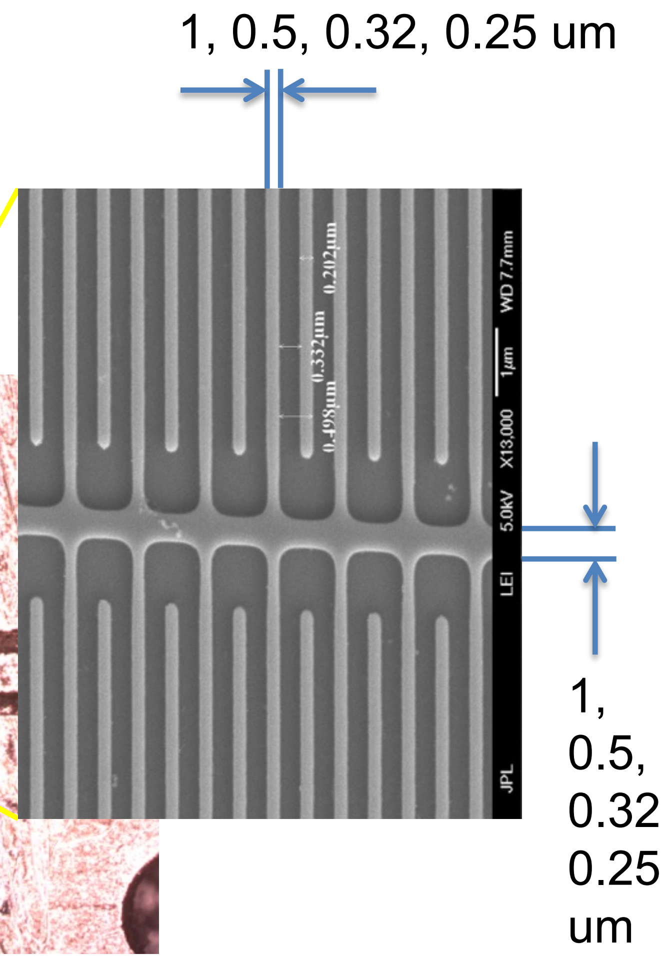
- Measured gain of a prototype device (f_{pump} = 8.5 GHz)
- Pump power ~ 100 uW

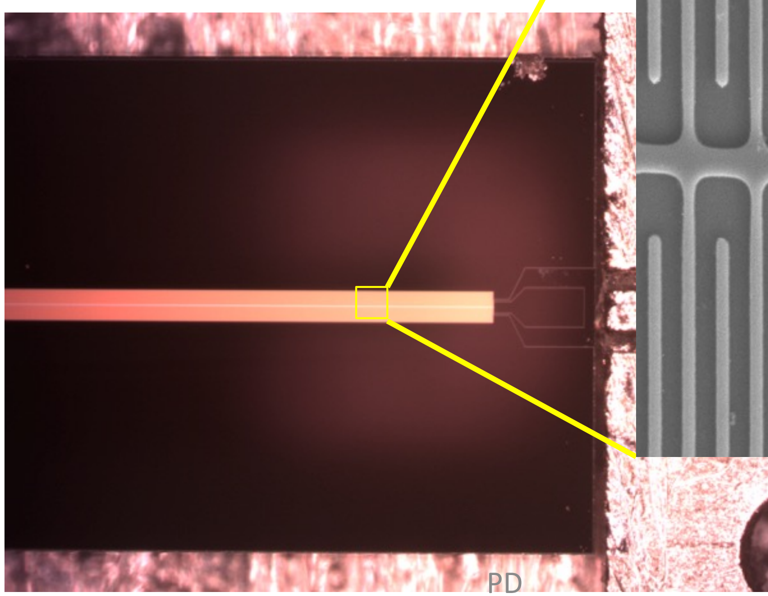


- Compare to cavity paramp with ~1 - 10MHz bandwidth

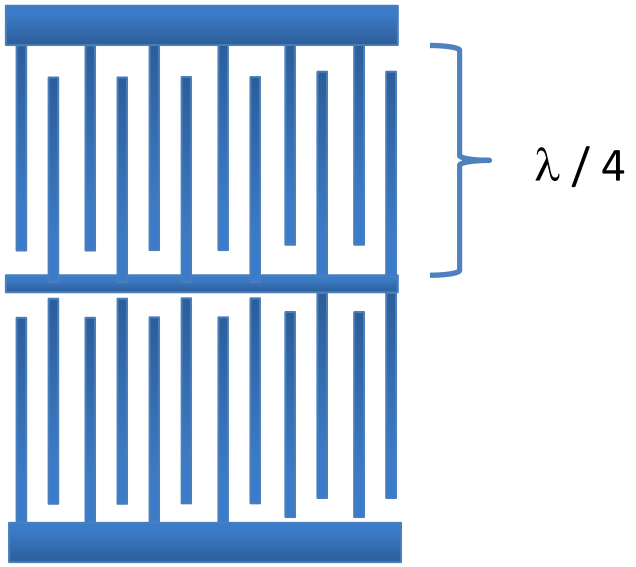
50 ohm KIT

- Z_0 = 200-300 ohms for CPW KIT (due to large L_{kin})
- Increase capacitance with IDC
- v_{ph} = 0.004 - 0.016 c
- 2.5 cm length
- -18 to -22 dB return loss from TDR
- Other 50Ω KITs: AA Adamyan et al. (2016), Chaudhuri et al. (2017).

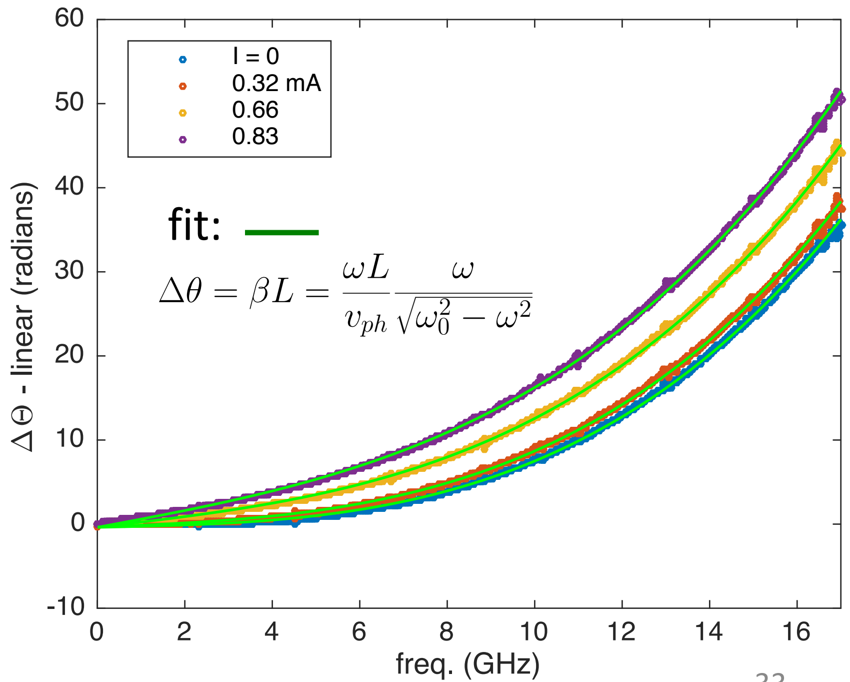




50 ohm KIT



- Lines are dispersive
  - λ / 4 resonance from IDC finger length
  - f_{res} = 60 GHz for 0.5 um line width
  - Pump harmonics suppressed due to phase mismatch
    - no additional stop bands needed
  - Effect on bandwidth, phase matching



Three-wave vs Four-wave Mixing

4 WM:

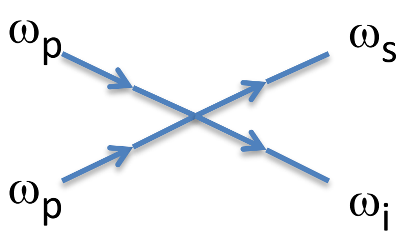
$$L(I) = L_0\left(1+\frac{I^2}{I_*^2}\right)$$

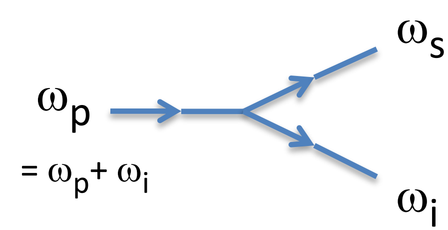
3 WM:

$$L(I) = L_0\left(1+\frac{I^2+2II_{DC}+I_{DC}^2}{I_*^2}\right)$$

$$\frac{\partial^2 I}{\partial z^2} - \frac{\partial}{\partial t}\left[L(I)C\frac{\partial I}{\partial t}\right] = 0$$

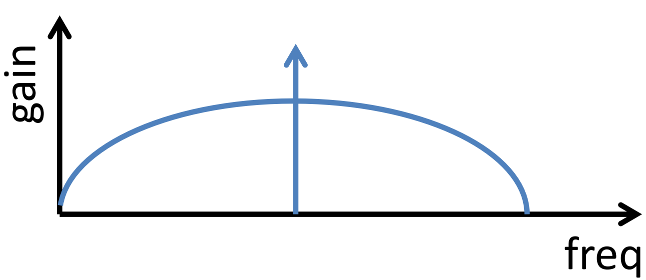
$2\omega_p = \omega_p + \omega_i$

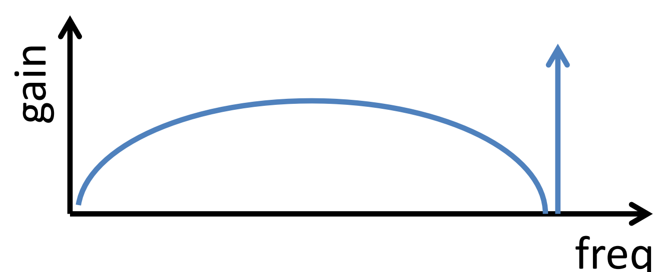




$$G_s = \exp\left(g 2\pi L/\lambda\right) \qquad g = \frac{1}{8}\frac{I_{pump}^2}{I_*^2} \quad /\mathrm{rad}$$

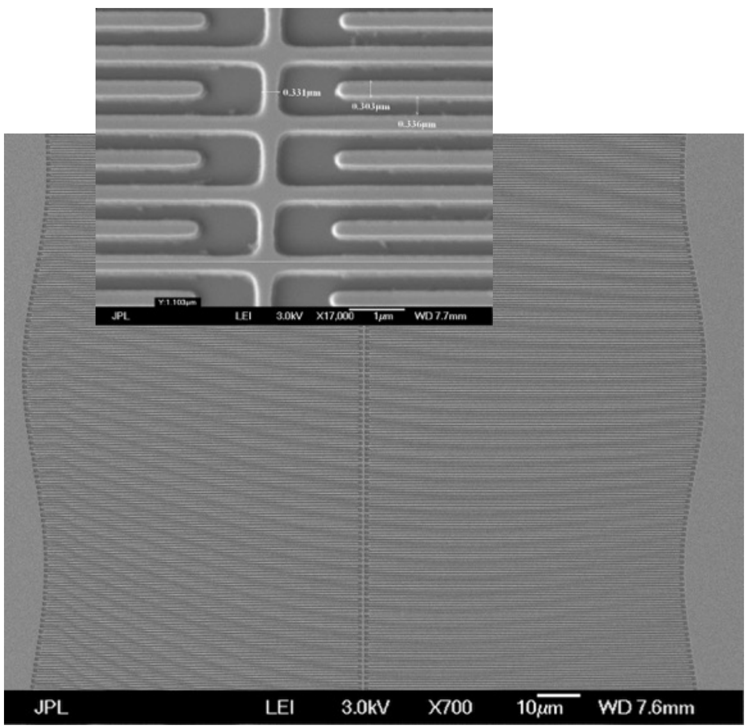
$$g = \frac{1}{4}\frac{I_{DC}I_{pump}}{I_*^2} \quad /\mathrm{rad}$$

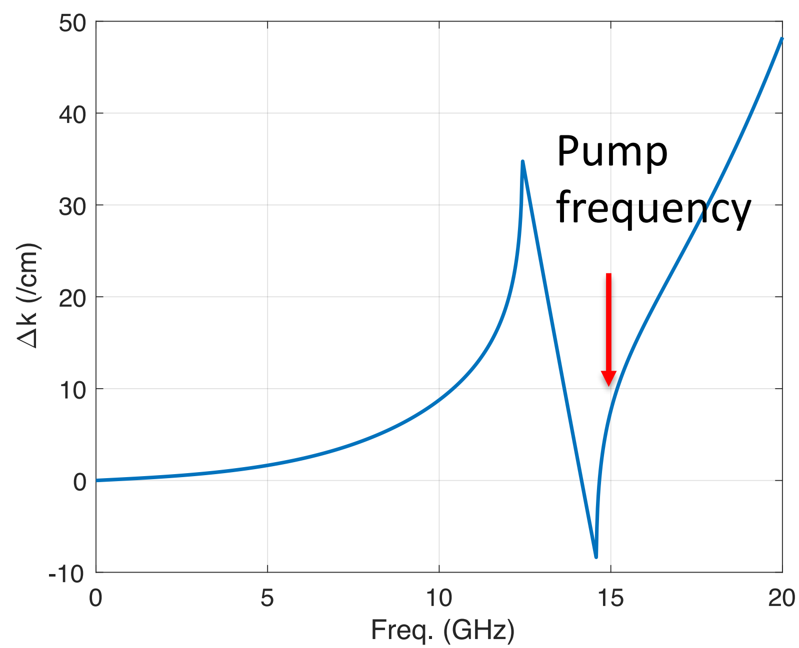




3WM: Engineering dispersion

- Dispersive effect of capacitor finger self-resonance produces phase mismatch between pump and signal/idler
  - Can compensate by adding periodic modulation

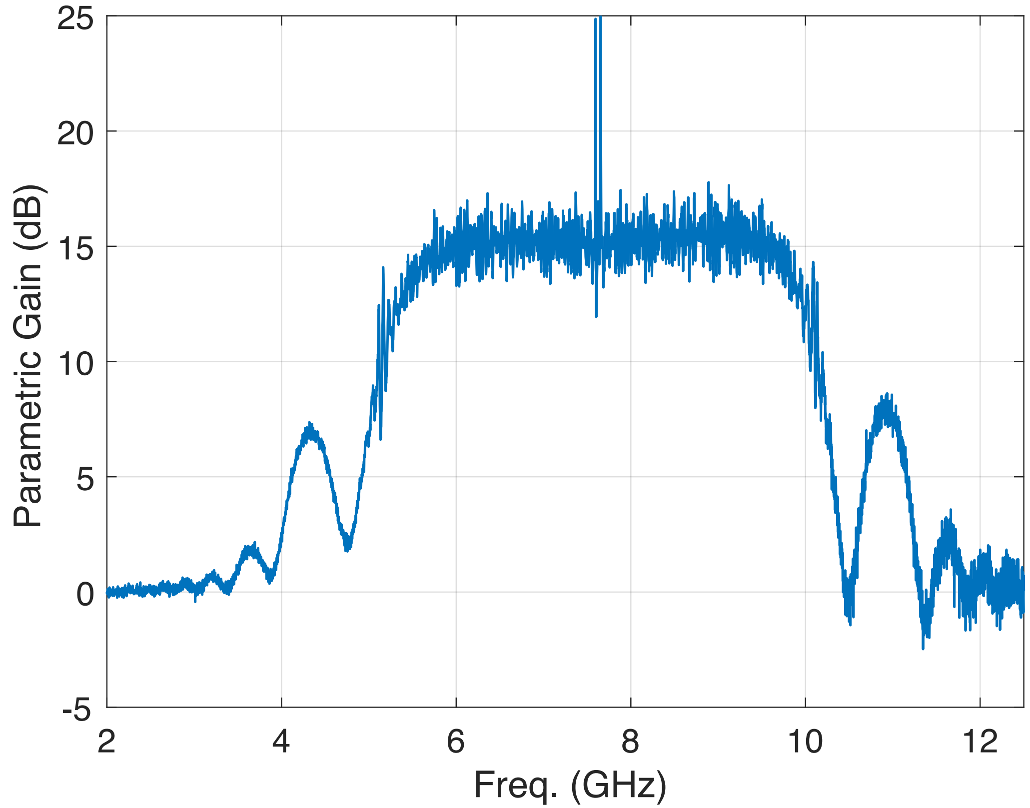




Gain

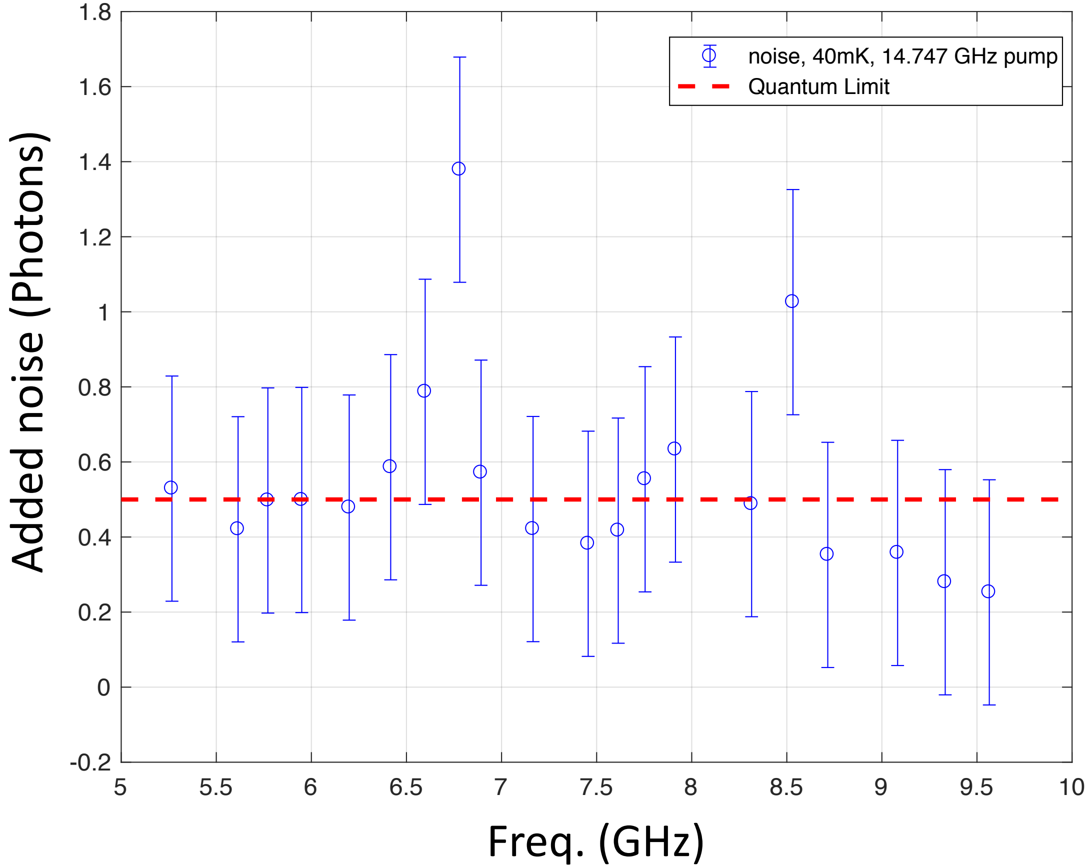
- 320 nm linewidth
- 2.5 cm total length
- Pump power in range 200 nW to 2 μW
  - Depends on applied DC current

Pump: 15.25 GHz



Noise

- Y-factor noise measurement

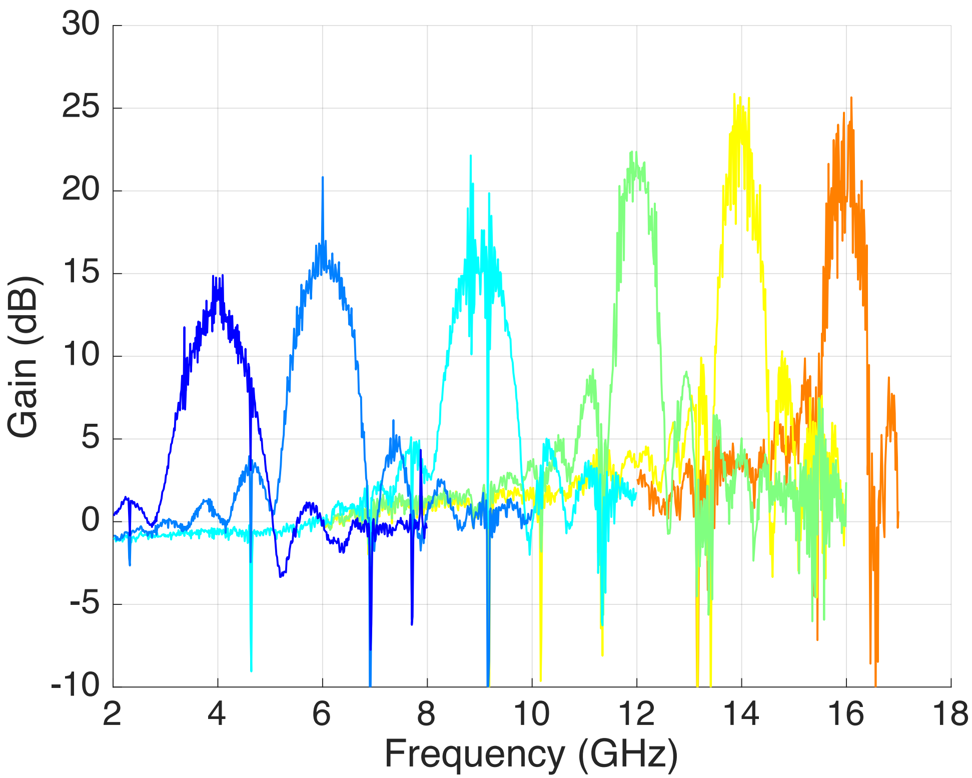


Conclusions

- Kinetic inductance can provide a purely reactive nonlinearity
- Important parameter for MKID detectors
- Can be used to make interesting devices
  - Up-converter/ Current sensor: KPUP
  - Parametric Amplifiers
  - Squeezed state generators
  - Multipliers

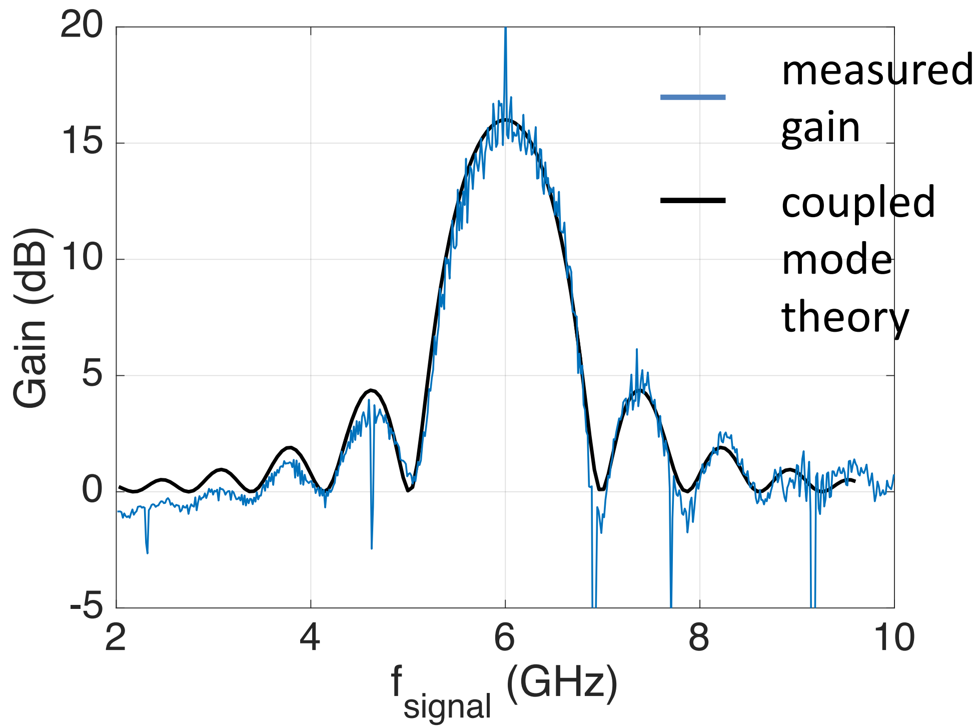
4WM in 0.25 micron linewidth devices, 2.5 cm length

- Non-phase matched 4 wave mixing
- Pump frequency is not constrained
  - No dispersion engineering
  - not phase matched
- 15 – 20 dB
- Pump power ~ 5 uW
- relatively narrow bandwidth due to dispersion



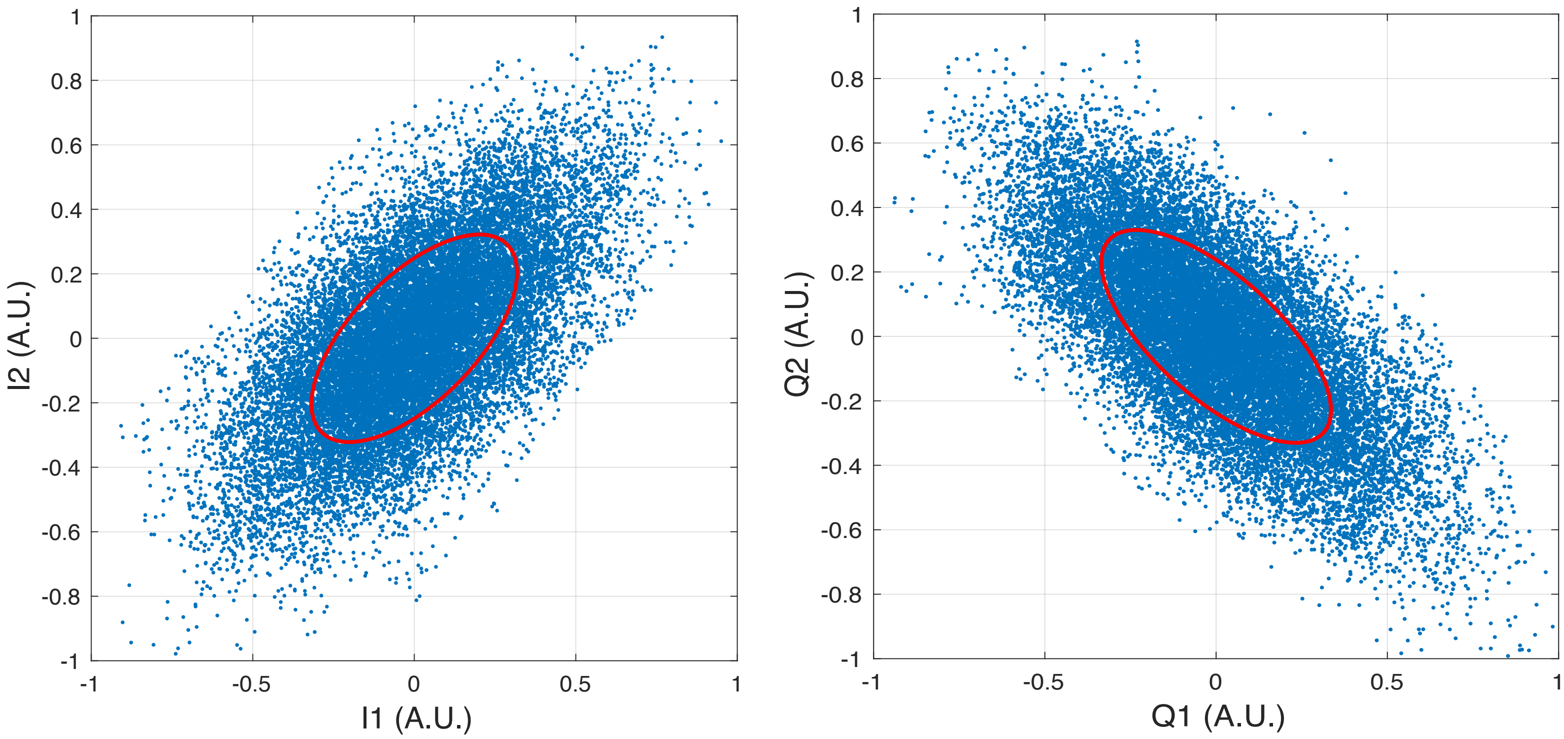
Gain measurement compared to theory

- Coupled mode theory including harmonic generation and measured dispersion
  - Single parameter fit : $(I / I_*)^2 = 0.08$
  - Max nonlinearity agrees with phase shift vs. dc current measurements



Signal-idler noise correlation

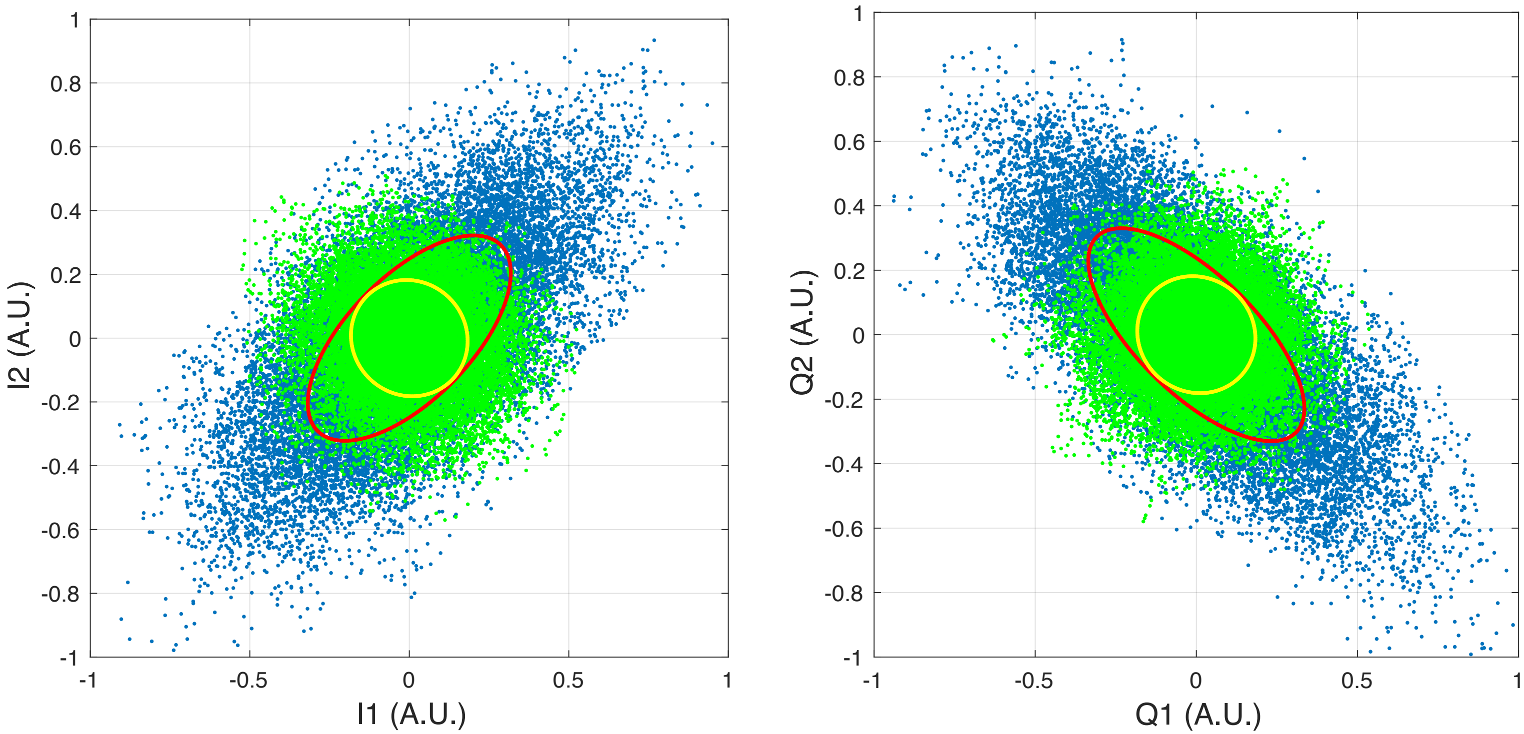
- Noise at frequencies symmetrically located around $f_{pump}/2$: 7.1384 GHz (I1, Q1), 7.6086 GHz (I2, Q2)



Pump on

Signal-idler noise correlation

- Noise at frequencies symmetrically located around $f_{pump}/2$: 7.1384 GHz (I1, Q1), 7.6086 GHz (I2, Q2)



Pump on

Pump off